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Junior High School Students' Misconceptions and Problem-Solving Strategies in Relations and Functions

Marwati Abd Malik[1], Mas'ud B[1], Arsy Handayani[1*], Dewi Yuniarti Bayu[1], Abdul Salam Yusuf[2]

[1]Mathematics Teacher Professional Education Program, Faculty of Teacher Training and Education, Universitas Muhammadiyah Parepare, South Sulawesi, Indonesia
[2]SMP Negeri 2 Parepare, South Sulawesi, Indonesia

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ABSTRACT

Mathematical misconceptions can hinder students' problem-solving abilities, particularly in relations and functions, which involve multiple representations. This study aimed to identify misconceptions and mathematical difficulties and analyze the problem-solving strategies of eighth-grade students at SMP Negeri 2 Parepare in relations and functions by integrating the Certainty of Response Index (CRI) with an analysis based on Polya's problem-solving stages. A qualitative approach with a descriptive-narrative design was employed. Twenty-eight students completed a CRI-based diagnostic test, after which six students were purposively selected based on their test scores, with two students representing each of the high-, medium-, and low-score categories. These six participants completed a written test consisting of non-routine, open-ended problems structured around Polya's stages. Five of them subsequently participated in semi-structured interviews to clarify aspects of their responses at stages requiring further investigation. Data were analyzed using the Miles and Huberman model. Trustworthiness was established through source triangulation, involving comparisons across participants, and methodological triangulation, involving comparisons among CRI results, written responses, and interviews. The findings revealed conceptual difficulties in distinguishing between the codomain and range and understanding the conditions that define a function, representational errors involving the use of set labels in ordered pairs, and procedural difficulties in applying formulas to tabulated data. At the understanding-the-problem stage, students generally identified the domain correctly but were less accurate in identifying the range. At the devising-a-plan stage, some students made notational errors. At the carrying-out-the-plan stage, difficulties were procedural. At the looking-back stage, most students checked only whether pairings existed without verifying their uniqueness.

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Corresponding Author: Arsy Handayani | ✉ arsyhandayani44@gmail.com

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Introduction

Among the mathematics topics taught at the junior high school level, relations and functions require conceptual understanding and the ability to connect multiple mathematical representations. These representations include arrow diagrams, ordered pairs, tables, and Cartesian graphs, each of which depicts relationships between two sets. Mastering relations and functions requires not only proficiency in computational procedures but also an understanding of the domain, codomain, range, and conditions under which a relation qualifies as a function. Difficulties in connecting these concepts and representations may lead students to produce incorrect answers even when they can perform certain mathematical procedures. Jader et al. (2020) reported that mathematics textbooks across various countries tend to be dominated by routine problems, limiting students' opportunities to engage with non-routine problems that require deeper conceptual understanding. Pratiwi (2022) also highlighted the need to address misconceptions about functions, as students' misunderstandings are not always evident in their final answers.

Mathematical misconceptions are an important concern in mathematics education because they reflect understandings that are inconsistent with accepted mathematical concepts. In this study, misconceptions are defined as students' understandings or ideas that conflict with such concepts. They may develop through prior learning experiences, flawed intuitive reasoning, or instruction that prioritizes procedural proficiency over conceptual understanding. In relations and functions, misconceptions may arise when students equate relations with functions, fail to distinguish among the domain, codomain, and range, or misunderstand the requirement that each element of a function's domain must be mapped to exactly one element of its codomain. Such misconceptions can affect problem solving because conceptual understanding underpins students' ability to identify relevant information, select strategies, execute procedures, and verify solutions. Identifying misconceptions therefore requires examining not only answer accuracy but also students' confidence in their responses and the reasoning underlying them.

The Certainty of Response Index (CRI) offers an approach to examining students' conceptual understanding and confidence in their answers. It combines answer accuracy with students' reported confidence, helping distinguish among students who understand a concept, those who lack understanding, and those who hold misconceptions (Hasan et al., 1999). CRI is relevant to relations and functions because students may give incorrect answers with varying levels of confidence. However, diagnostic testing alone is insufficient to explain how the identified misconceptions affect students' problem-solving strategies. A more comprehensive understanding requires connecting the analysis of misconceptions with an examination of the mathematical problem-solving process.

This process can be examined using Polya's four stages: understanding the problem, devising a plan, carrying out the plan, and looking back. These stages provide a framework for investigating how students interpret information, select strategies, apply procedures, and evaluate their solutions. Lestari & Rosdiana (2021) emphasized the importance of systematically examining students' errors to identify the underlying difficulties in their solution processes. In relations and functions, for example, students who do not understand the conditions that define a function may struggle to select an appropriate strategy for representing a relation or to determine whether an arrow diagram satisfies the definition of a function. An analysis based on Polya's stages can thus provide insight into the relationship between students' conceptual understanding and the problem-solving strategies they use.

Previous studies have examined students' misconceptions and problem-solving abilities in relations and functions, although with different emphases. Afgani et al. (2022) used a four-tier diagnostic test to identify junior high school students' misconceptions, focusing on errors in classifying relations and functions. Kefi et al. (2021) used CRI to analyze students' difficulties in solving problems involving relations, while Pratiwi (2022) investigated students' misconception profiles concerning functions through diagnostic testing. These studies describe the forms and characteristics of students' conceptual difficulties but do not specifically examine how the identified misconceptions relate to their problem-solving strategies at each of Polya's stages. By contrast, Nisa & Asmarani (2023) and Idris & Imami (2024) examined students' mathematical problem-solving abilities using Polya's stages. Their studies describe students' performance at each stage but do not specifically connect that performance with misconception categories identified through diagnostic instruments. Arfany & Faradiba (2022), Listiana & Rohmah (2024), and Saputri et al. (2023) further contributed to understanding students' conceptual, representational, and reasoning errors in relations and functions. Collectively, these studies provide an important foundation for understanding misconceptions and problem-solving abilities, but the relationship between the two requires a more integrated examination.

This gap highlights the need for research that not only identifies misconceptions but also investigates how they manifest in and influence students' strategies at each stage of problem solving. The distinctive contribution of this study lies in integrating CRI-based misconception analysis with an examination of problem-solving strategies using Polya's stages. This integration enables the researchers to connect answer accuracy and confidence levels with the thinking processes reflected in students' written responses and further clarified through interviews. Rather than merely documenting conceptual, representational, and procedural errors, the study examines the misconceptions underlying these errors and their implications for understanding the problem, devising a plan, carrying out the plan, and looking back. Theoretically, the study seeks to clarify the relationship between the characteristics of students' misconceptions and their mathematical problem-solving strategies. Practically, its findings may inform the design of diagnostic assessments and instruction that emphasize conceptual understanding, multiple representations, and solution verification.

Identifying students' misconceptions and mathematical difficulties provides a diagnostic foundation, while examining their strategies at each of Polya's stages offers insight into how these misconceptions manifest during problem solving. Integrating these perspectives can provide a more comprehensive basis for teaching relations and functions to eighth-grade students at SMP Negeri 2 Parepare. Accordingly, this study aims to: (1) identify the forms of misconceptions and mathematical difficulties experienced by eighth-grade students in relations and functions; and (2) analyze their mathematical problem-solving strategies using Polya's stages, with particular attention to the misconceptions underlying their solution processes.

Method

Research Design

This study employed a qualitative approach with a descriptive-narrative design. A qualitative approach was selected to explore students' misconceptions, thinking processes, and mathematical problem-solving strategies in relations and functions in depth. The descriptive-narrative design was used to describe students' problem-solving processes based on their written responses and interview accounts, encompassing understanding the problem, devising

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https://doi.org/10.51574/kognitif.v6i4.6073 Volume 6, Issue 4, October - December 2026, pp. 1828 –1847

a plan, carrying out the plan, and looking back. Rather than statistically testing hypotheses, the study sought to develop a contextual understanding of students' conceptual understanding and difficulties (Creswell, 2014; Moleong, 2019).

Participants

The study was conducted at SMP Negeri 2 Parepare during the second semester of the 2025/2026 academic year. Class VIII.5 comprised 33 students, of whom 28 completed and submitted a diagnostic test incorporating the Certainty of Response Index (CRI). Based on the results of these 28 students, six were purposively selected for further investigation: two from the high-performance category (S1 and S6), two from the medium-performance category (S2 and S3), and two from the low-performance category (S4 and S5). Students were grouped according to the number of correct responses to the ten diagnostic test items: scores of 8–10 were classified as high, 5–7 as medium, and 0–4 as low. In this article, these performance categories refer specifically to students' achievement on the diagnostic test rather than to their general mathematical ability. CRI scores, meanwhile, were used to identify students' confidence in their answers. Test performance and response confidence were therefore analyzed as separate dimensions. Participant selection also considered the relevance of students' response patterns to the research focus and their willingness to complete the written test and, for five participants, participate in interviews. Participants were assigned codes S1–S6 to protect their identities.

Instruments

The researcher served as the primary instrument, undertaking data collection, processing, and interpretation. Supporting instruments comprised a CRI-based diagnostic test, a written test of non-routine problems, and a semi-structured interview guide. The CRI-based diagnostic test comprised ten multiple-choice items on relations and functions, each with four answer options (A–D). The items assessed understanding of relations and functions; identification of the domain, codomain, and range; and representations of relations and functions. Correct answers received one point, and incorrect answers received zero points, yielding total scores ranging from 0 to 10. For each answer, students indicated their confidence on a scale from 0 to 5. CRI scores of 0–2 were classified as low confidence, whereas scores of 3–5 were classified as high confidence. Students' understanding was interpreted by combining answer accuracy with confidence levels. Correct answers with high CRI scores indicated conceptual understanding, incorrect answers with low CRI scores indicated a lack of conceptual understanding, and incorrect answers with high CRI scores indicated potential misconceptions (Hasan et al., 1999). Correct answers with low CRI scores were treated as low-confidence correct responses and were not automatically interpreted as evidence of secure conceptual understanding. The researcher developed the instrument based on the learning indicators for relations and functions, using the 0–5 confidence scale described by Hasan et al. (1999). The instrument had not undergone formal validation by experts or teachers.

The written test comprised four non-routine problems developed around Polya's problem-solving stages: understanding the problem, devising a plan, carrying out the plan, and looking back. The first problem assessed students' ability to identify the domain, codomain, and range and determine whether a relation was a function. The second assessed their ability to represent a relation using ordered pairs and an arrow diagram and explain their chosen strategy. The third assessed their ability to calculate function values, present the results in a table, and identify

patterns in the relationship. The fourth assessed their ability to evaluate a statement against the defining conditions of a function and formulate an appropriate conclusion. Responses were analyzed qualitatively, focusing on conceptual understanding, strategy selection, procedural accuracy, and students' ability to check their solutions. Each response was examined using indicators corresponding to Polya's stages and the types of errors observed, without treating every error as a misconception. The semi-structured interview guide was used to explore students' reasoning, considerations, and thinking processes while solving the problems. Interview questions were tailored to each participant's written responses to clarify solution steps, reasons for selecting particular strategies, conceptual understanding, and confidence in their answers. Interviews focused on stages for which the written responses required clarification; consequently, the stages covered varied across participants.Examples of items from each instrument are presented in Table 1.

Table 1. Examples of Research Instrument Items

Instrument	Focus and format	Example item
CRI-based diagnostic test	Ten multiple-choice items (A–D) addressing the classification of relations and functions, representations, and conceptual understanding. Each item included a request for justification and a CRI confidence rating of 0–5.	Item 2. Consider the ordered pairs $\{(1,a),(1,b),(2,c)\}$. Which statement is correct? A. The relation is a function. B. The relation is not a function because the codomain is different. C. The relation is not a function because one domain element is paired with two elements. D. The relation is a function because all elements have a corresponding element. Reason: CRI confidence rating: 0 / 1 / 2 / 3 / 4 / 5
Written test of non-routine problems	Four constructed-response problems, each representing one of Polya's stages (MM, MS, MR, and MK). Responses were analyzed in terms of both the solution process and the final answer.	Item 4: Looking back. A student claims that the following relation is a function: $\{(1,a),(2,b),(2,c),(3,d)\}$. a. Do you agree with the student's claim? Explain your answer. b. Re-examine the relation using the defining conditions of a function. c. State the correct final conclusion.
Semi-structured interview guide	Opening questions, questions corresponding to Polya's stages for each problem, and closing questions. Follow-up questions were developed based on participants' written responses.	Item 1: Understanding the problem. "What do you understand by the term domain?"; "What conditions must a relation satisfy to be a function?" Item 4: Looking back. "How do you check whether the relation is a function?"; "Why are the pairs $(2,b)$and $(2,c)$important when checking this relation?"

Data Collection

Data were collected on two occasions. The CRI-based diagnostic test was administered to students in Class VIII.5 on May 18, 2026. On May 22, 2026, the six selected participants completed the written test of non-routine problems. After the test, semi-structured interviews were conducted with five participants (S1–S5) on the same day. The interviews were informed by a review of the written responses and focused on aspects requiring clarification rather than

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https://doi.org/10.51574/kognitif.v6i4.6073 Volume 6, Issue 4, October - December 2026, pp. 1828 –1847

covering all four of Polya's stages for every participant. Questions were tailored to each student's responses to clarify solution steps, reasons for strategy selection, conceptual understanding, and confidence in their answers.

The interview coverage varied across participants. S1 was interviewed about understanding the problem and looking back. S2 was interviewed about understanding the problem and devising a plan, with additional clarification of the defining conditions of a function at the looking-back stage. S3 was interviewed about understanding the problem and looking back, whereas S4 and S5 were interviewed about carrying out the plan. S6 was not interviewed; therefore, interpretations of this participant's responses were based on the CRI results and written work. The data collection procedure is presented in Figure 1. The collected data comprised CRI diagnostic test results, written responses to the non-routine problems, and interview notes. These data were subsequently reduced and analyzed as described in the Data Analysis section.

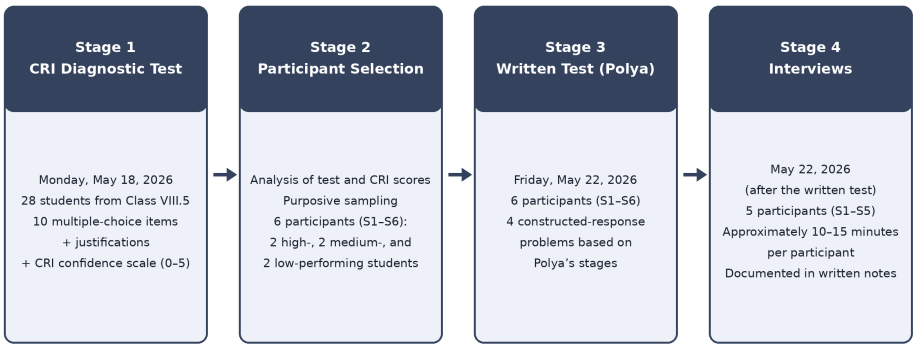


Figure 1. Data Collection Procedure

Data Analysis

Data were analyzed using the Miles and Huberman interactive model, comprising data reduction, data display, and conclusion drawing and verification. During data reduction, the researcher selected and organized diagnostic test results, written responses, and interview transcripts according to the research focus. Students' responses were then coded using Polya's four stages: understanding the problem (MM), devising a plan (MS), carrying out the plan (MR), and looking back (MK). Errors and difficulties were examined using conceptual, notational/representational, and procedural categories. These categories provided an analytical framework, but not every error was automatically classified as a misconception. The classification of a finding as a misconception took into account answer accuracy, confidence levels indicated by CRI scores, and, where available, explanations provided during interviews. Incorrect answers with CRI scores of 3–5 were classified as indications of misconceptions, whereas incorrect answers with CRI scores of 0–2 were classified as indicating a lack of conceptual understanding. Correct answers with CRI scores of 0–2 were treated as low-confidence correct responses. Items that were unanswered or lacked a CRI score were not classified. When the evidence indicated difficulty in executing solution steps without an underlying conceptual misunderstanding, the finding was described as a procedural difficulty rather than automatically classified as a misconception.

During data display, the findings were organized into descriptive accounts, cross-participant comparison tables, and relevant examples of students' written work. The

relationship between misconceptions and problem-solving strategies at each of Polya's stages was examined through five steps: (1) Classifying CRI test responses into conceptual understanding, guessing, lack of conceptual understanding, and indications of misconceptions; (2) Coding written responses and interview transcripts according to Polya's stages (MM, MS, MR, and MK); (3) Identifying the type of error at each stage—conceptual, notational/representational, or procedural—and determining whether it met the criteria for a misconception; (4) Constructing a matrix linking participants, Polya's stages, and error types; and (5) Interpreting how these errors influenced strategy selection at each stage through comparisons across participants and diagnostic test performance levels.

The final phase involved drawing and verifying conclusions by comparing diagnostic test results, written responses, and interview data where available (Miles et al., 2014). Trustworthiness was examined through source and methodological triangulation. Source triangulation involved comparing information across the six participants. Methodological triangulation involved comparing CRI diagnostic test results, written responses, and interview data for participants and stages for which all three data sources were available. Because interviews were conducted with only five participants and covered selected stages, methodological triangulation was partial. Findings concerning S6 and stages not covered in the interviews were based on CRI results and written responses.

Research Findings

Twenty-eight of the 33 students in Class VIII.5 at SMP Negeri 2 Parepare completed the Certainty of Response Index (CRI)-based diagnostic test on May 18, 2026. Subsequently, on May 22, 2026, six selected students completed the written test of non-routine problems, and five of them (S1–S5) were interviewed after completing the test on the same day. Semi-structured interviews were used to clarify responses requiring further explanation. S6 was not interviewed. The six participants comprised two students from each of the high-, medium-, and low-performance categories.

Table 2. Participant Characteristics

No.	Participant code	Diagnostic test score	Performance category
1	S1	9	High
2	S2	6	Medium
3	S3	6	Medium
4	S4	2	Low
5	S5	3	Low
6	S6	8	High

Note. Performance categories were determined using scores on the ten-item diagnostic test: high = 8–10, medium = 5–7, and low = 0–4.

The CRI classifications for the six participants are presented in Table 3. Each response was classified by combining answer accuracy with the confidence level reported for that item.

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Table 3. CRI Response Classifications by Participant

Participant	Correct answers	Conceptual understanding	Guessing	Lack of conceptual understanding	Indications of misconceptions	Unanswered items	Missing CRI ratings
S1	9	0	3	0	1	0	6
S2	6	1	5	4	0	0	0
S3	6	1	5	3	1	0	0
S4	2	2	0	2	4	0	2
S5	3	3	0	3	4	0	0
S6	8	8	0	0	1	1	0

Note. Conceptual understanding = a correct answer with a CRI rating of 3–5; guessing = a correct answer with a CRI rating of 0–2; lack of conceptual understanding = an incorrect answer with a CRI rating of 0–2; indications of misconceptions = an incorrect answer with a CRI rating of 3–5. Unanswered items and responses with missing CRI ratings were excluded from these four interpretive categories. Each participant completed a ten-item test.

As shown in Table 3, S4 and S5 had the highest number of responses indicating misconceptions, with four items each. S1, S3, and S6 each had one such response, whereas S2 had no incorrect responses accompanied by high confidence. These findings should be interpreted alongside the categories of guessing, lack of conceptual understanding, unanswered items, and missing CRI ratings. In particular, six of S1's responses could not be classified because the CRI ratings were missing.

Analysis of Students' Thinking Processes Using Polya's Stages

Excerpts from participants' written responses at each of Polya's stages are presented in Figures 2–5. The excerpts were selected to illustrate the responses underlying the findings at each stage. Identifying information was omitted from the answer sheets.

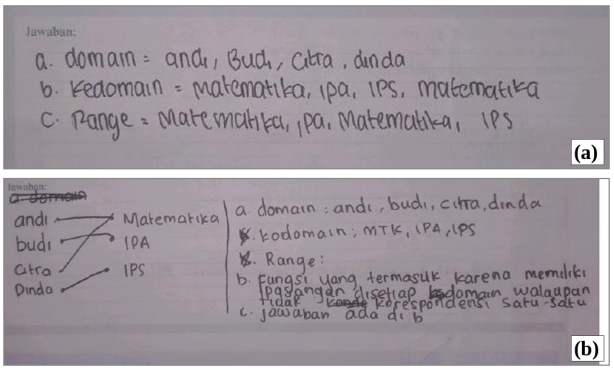
Understanding the Problem (MM)

At this stage, students were asked to identify the domain, codomain, and range of a relation presented in the context of borrowing subject-specific textbooks and then determine whether the relation was a function. Their written responses revealed differences in their understanding of the components of a relation. S1 identified the domain but listed the same subject more than once in both the codomain and the range. During the interview, S1 explained that Mathematics had been listed twice because two students had borrowed mathematics textbooks, reflecting the belief that the same set element could be listed repeatedly. S1 also acknowledged some confusion about the distinction between the codomain and the range. These findings indicate that S1 did not fully understand that repeated occurrences do not constitute distinct elements of a set.

Meanwhile, S2, S4, and S5 omitted the range from their written responses. During the interview, S2 expressed uncertainty about whether the range included all elements of set B or only those paired with elements of the domain. The reasons for S4's and S5's omissions were not explored through interviews. A different pattern emerged for S3. Although S3 omitted the range from the written response, the participant explained during the interview that the domain consisted of students' names, the codomain consisted of the available subjects, and the range consisted of the subjects paired with students. S3 further explained that the range and codomain contained the same elements in this problem because every subject in the codomain was paired

with a student. This finding reveals a discrepancy between the participant's written response and oral explanation. Omitting the range from a written response was therefore insufficient evidence to conclude that the participant did not understand the concept.

When explaining the conditions under which a relation qualifies as a function, S1 stated that a relation was a function if every element on the left had a corresponding element on the right, without mentioning that each domain element must have exactly one corresponding element. These findings identify difficulties in understanding the range and applying the complete definition of a function as important aspects of students' performance at the understanding-the-problem stage. Difficulties concerning the uniqueness of the corresponding element became more apparent at the looking-back stage. Figure 2 presents S1's and S2's written responses to Problem 1. S1 listed Mathematics twice in both the codomain and the range, whereas S2 identified the domain and codomain but omitted the range.



Domain: Andi, Budi, Citra, Dinda
Codomain: Mathematics, Natural Sciences, Social Sciences

It is a function because there is a corresponding element for every element in the codomain, even though it is not a one-to-one correspondence.

The answer is given in part b.

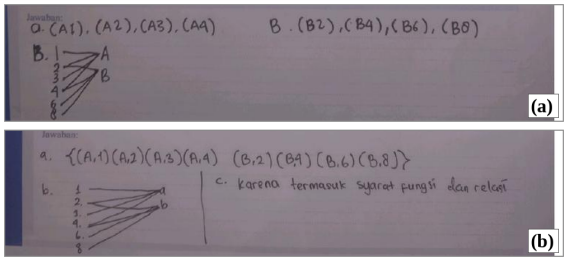
Figure 2. Written Responses at the Understanding-the-Problem Stage (Problem 1): (a) S1; (b) S2

Devising a Plan (MS)

At this stage, students were asked to represent a relation from set A to set B using ordered pairs and an arrow diagram and to explain their chosen strategy. Several participants' written responses contained errors in ordered-pair notation. S2 used the set label A as the first component of an ordered pair, regarding the set name as the first component and a number as the second. S1, S3, and S5 also used set labels when writing ordered pairs. During the interview, S2 explained that insufficient attention had been paid to the requirement that ordered pairs be formed from the elements being related. The reasons underlying S1's, S3's, and S5's notation were not explored through interviews. For these three participants, the findings were therefore limited to representational errors evident in their written work rather than confirmed explanations of their reasoning. S6 represented the ordered pairs more accurately as $\{(1,2),(2,4),(3,6),(4,8)\}$.

These findings indicate that devising a plan involves not only selecting a representation but also understanding the meaning of its constituent elements. Errors in notation can affect the accuracy with which students represent relationships between domain and codomain elements. Because the reasoning underlying the notation was clarified through an interview only for S2, interpretations concerning the other participants were based on written evidence and were not generalized as reflecting the same thinking process. Figure 3 presents the ordered pairs written by S2 and S3 for Problem 2. Both used a set label, A or B, as the first component of the ordered pairs rather than an element of set A.

Kognitif: Jurnal Riset HOTS Pendidikan Matematika
https://doi.org/10.51574/kognitif.v6i4.6073 Volume 6, Issue 4, October - December 2026, pp. 1828 –1847



Panel (a)
Answer:
a. (A1), (A2), (A3), (A4) B. (B2), (B4), (B6), (B8)
b. [Arrow diagram as shown in the original.]
Panel (b)
Answer:
a.
$\{(A,1),(A,2),(A,3),(A,4),(B,2),(B,4),(B,6),(B,8)\}$
b. [Arrow diagram as shown in the original.]
c. Because it satisfies the conditions for functions and relations.

Figure 3. Written Responses at the Devising-a-Plan Stage (Problem 2): (a) S2; (b) S3

Carrying Out the Plan (MR)

At this stage, students were asked to calculate $f(2)$and $f(5)$for the function $f(x) = 2x + 3$, present the function values for $x = 1,2,3,$and 4in a table, and identify the resulting pattern. Written responses, supplemented by interviews with S4 and S5, revealed variation in students' ability to execute computational procedures. Some students substituted values and performed calculations correctly, whereas others struggled to continue their calculations or apply the formula consistently when completing the table. S3 initially obtained $f(2) = 7$from $2(2) + 3$but then divided the result by 2 to obtain 3.5. The participant also left the table and subsequent parts of the problem incomplete. Because S3 was not interviewed about this stage, the reason for the additional division could not be established. Based on the written response, this finding is more appropriately interpreted as a procedural difficulty than as evidence of a persistent misconception.

In contrast, S4 carried out the calculations correctly. The participant explained how values of $x$were substituted into $f(x) = 2x + 3$to obtain $f(2) = 7$and $f(5) = 13$. S4 also correctly calculated the function values for $x = 1,2,3,$and 4as 5, 7, 9, and 11, respectively, consistently applying the same procedure when completing the table. S5 likewise explained the substitution procedure and correctly obtained $f(2) = 7$and $f(5) = 13$. However, S5 acknowledged a lack of care in calculating and recording results and expressed uncertainty about the accuracy of the values entered in the table. S6 exhibited a different difficulty. Although the participant correctly obtained $f(2) = 7$and $f(5) = 13$, the table contained the expressions $1x$, $2x$, $3x$, and $4x$, indicating that the function formula had not been fully applied. Because S6 was not interviewed, the reasoning underlying this response could not be confirmed. The finding was therefore interpreted as inconsistent application of the formula in the table, based on the written evidence. Thus, difficulties at the carrying-out-the-plan stage took different forms: an unnecessary additional calculation by S3, insufficient care by S5, and inconsistent application of the formula by S6. Figure 4 presents S3's and S6's responses to Problem 3. S3 divided the correct result, $f(2) = 7$, by 2 to obtain 3.5 and left the table blank. S6 calculated $f(2)$and $f(5)$correctly but entered $1x$, $2x$, $3x$, and $4x$in the $f(x)$column.

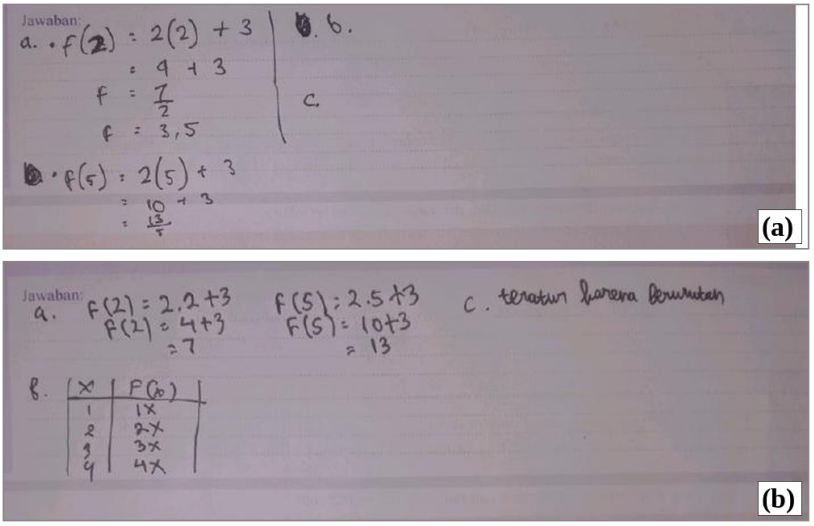


Figure 4. Written Responses at the Carrying-Out-the-Plan Stage (Problem 3): (a) S3; (b) S6

Looking Back (MK)

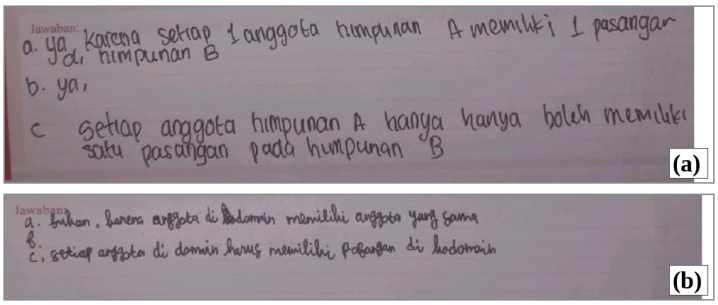
At this stage, students were asked to evaluate the claim that the relation $\{(1,a),(2,b),(2,c),(3,d)\}$was a function. According to the definition of a function, this relation is not a function because the domain element 2 is paired with two distinct elements, $b$and c. Students' written responses revealed difficulties in checking whether each domain element had exactly one corresponding element. S1 concluded that the relation was a function because every domain element had a corresponding element. During the interview, S1 identified the pairs linking 1 to a, 2 to $b$and c, and 3 to d, but still regarded the relation as a function because every number had a corresponding element. The participant did not account for the fact that 2 was paired with two elements. S4's written response likewise classified the relation as a function without accounting for the two arrows originating from the domain element 2; the reasoning behind this answer was not explored through an interview.

S3 initially checked whether every number had an arrow without considering whether each number had only one arrow. Following additional questions, however, S3 explained that the relation was not a function because the domain element 2 was paired with two distinct elements, $b$and c. S5 also classified the relation as a function in the written response, but the reason could not be confirmed because the participant was not interviewed about this stage. During the interview, S2 stated a belief that a function had to be one-to-one but remained uncertain about whether a relation in which one element had more than one corresponding element could still be a function. S6 correctly classified the relation as not being a function in the written response, but the reasoning underlying this conclusion was not clarified through an interview. The analysis therefore distinguished among written responses, initial oral explanations, and responses elicited through follow-up questions.

These findings indicate that some students did not check their solutions comprehensively. They tended to verify the existence of corresponding elements without consistently checking whether each domain element had exactly one corresponding element. S3's ability to revise the explanation following probing demonstrates the value of interviews in clarifying students' understanding. However, a response produced after probing cannot be equated with independent checking during the written test. Figure 5 presents responses to Problem 4. S1 stated that the relation was a function because each element of set A had one corresponding

Kognitif: Jurnal Riset HOTS Pendidikan Matematika
https://doi.org/10.51574/kognitif.v6i4.6073 Volume 6, Issue 4, October - December 2026, pp. 1828 –1847

element in set B. S6 stated that the relation was not a function, but the written justification referred to codomain elements rather than identifying the domain element 2 as having two corresponding elements. Consequently, S6's response could not be confidently interpreted as evidence of checking the uniqueness requirement.



Panel (a)
Answer:
a. Yes, because each element of set A has one corresponding element in set B.
b. Yes.
c. Each element of set A may have only one corresponding element in set B.

Panel (b)
Answer:
a. No, because the elements in the codomain have the same corresponding element.
b. [Left blank.]
c. Every element in the domain must have a corresponding element in the codomain.

Figure 5. Written Responses at the Looking-Back Stage (Problem 4): (a) S1; (b) S6

Summary of Findings Across Polya's Stages

Table 4 summarizes the findings at each of Polya's problem-solving stages based on written responses, CRI diagnostic test results, and interviews conducted with five participants at selected stages. The table includes participants' demonstrated competencies, difficulties, and strategies rather than misconceptions alone.

Table 4. Summary of Participant Findings Across Polya's Stages

Participant	Understanding the problem (MM)	Devising a plan (MS)	Carrying out the plan (MR)	Looking back (MK)
S1	Identified the domain but repeated elements in the codomain and range. During the interview, considered repeated listing of the same element acceptable and expressed confusion about the codomain and range. Considered the existence of a corresponding element for every domain element sufficient for a relation to be a function.	Used set labels in ordered pairs. The reason for this notation was not explored through an interview.	This stage was analyzed using written responses only. No stage-specific interview clarification was available; interpretation was limited to the written evidence.	Considered the relation a function if every domain element had a corresponding element, without checking the requirement of exactly one corresponding element.
S2	Identified the domain and codomain but omitted the range because its distinction from the codomain was not understood.	Used A as the first component of ordered pairs. During the interview, explained that the set name	During the interview, stated a belief that a function had to be one-to-one but remained uncertain about whether one element could have	Remained uncertain about the distinction between a function and a one-to-one correspondence and did not establish the requirement that every

Participant	Understanding the problem (MM)	Devising a plan (MS)	Carrying out the plan (MR)	Looking back (MK)
		was regarded as the first component.	more than one corresponding element.	domain element have exactly one corresponding element.
S3	Omitted the range from the written response but explained the domain, codomain, and range during the interview.	Used set labels incorrectly in ordered pairs, as shown in the written response. The reason was not clarified through an interview.	Obtained $f(2) =$ 7but then divided by 2 to obtain 3.5. Left the table and subsequent parts incomplete. The reason for this step was not confirmed through an interview.	Initially checked only for the existence of arrows but explained after probing that one domain element could not have two distinct corresponding elements.
S4	Identified the domain and codomain but omitted the range. The reason was not confirmed through an interview.	Exhibited representational errors in the written response. The reason for the notation was not confirmed through an interview.	Correctly and consistently substituted values, calculated function values, and completed the table.	Classified the relation as a function without accounting for the two arrows from the domain element 2. The reason was not explored through an interview.
S5	Identified the domain and codomain but omitted the range. The reason was not confirmed through an interview.	Used set labels in ordered pairs, as shown in the written response.	Correctly performed substitution and explained the calculations but reported insufficient care and uncertainty about the table entries.	Classified the relation as a function in the written response. The reason was not explored through an interview.
S6	Identified the components of the relation in the written response.	Represented the ordered pairs correctly.	Calculated function values correctly but entered $1x$, $2x$, $3x$, and $4x$in the table. The reason could not be confirmed because no interview was conducted.	Classified the relation as not being a function in the written response. The underlying reasoning was not clarified through an interview.

Identified Misconceptions and Difficulties

The analysis of written responses and interviews with five participants identified eight forms of misconceptions and difficulties, grouped into conceptual, notational/representational, and procedural categories, as presented in Table 5. Not all were classified as misconceptions; some were categorized as difficulties because evidence concerning students' confidence had not been confirmed.

Kognitif: Jurnal Riset HOTS Pendidikan Matematika
https://doi.org/10.51574/kognitif.v6i4.6073 Volume 6, Issue 4, October - December 2026, pp. 1828 –1847

Table 5. Identified Misconceptions and Difficulties

No.	Form of misconception or difficulty	Participants	Analytical category	Evidence sources
1	Misunderstanding the requirement that each domain element must have exactly one corresponding element	S1, S3, S4, S5	Conceptual (MisC/indications of difficulty)	Written responses; interviews with S1 and S3. CRI results for related indicators were interpreted separately.
2	Difficulty distinguishing between the codomain and the range	S1 and S2: confirmed through interviews; S4 and S5: omitted the range, with no interview confirmation	Conceptual difficulty	Written responses; interviews with S1 and S2
3	Using set labels as components of ordered pairs	S1, S2, S3, S5	Notational/representational (MisR)	Written responses; the reason for the notation was specifically confirmed for S2 through an interview
4	Repeatedly listing the same elements in the codomain and range	S1	Conceptual: understanding of set membership	Written response, interview with S1, and CRI Item 10
5	Errors in the procedure for calculating function values	S3	Procedural difficulty	Written response; this stage was not explored through an interview
6	Inconsistent application of the function formula when completing a table	S6	Procedural difficulty	Written response; this stage was not explored through an interview
7	Insufficient care and uncertainty about the values entered in a table	S5	Difficulty in executing procedures	Written response and interview with S5
8	Difficulty checking the defining conditions of a function using an arrow diagram	S1, S2, S3, S4, S5	Conceptual difficulty at the verification stage	Written responses; interviews with S1 and S3

Discussion

Misconceptions and Conceptual Difficulties in Relations and Functions

The findings indicate that students' difficulties with relations and functions extend beyond identifying the domain, codomain, and range to understanding the relationships among these three components. S1 repeatedly listed elements in the codomain and range, believing that each occurrence of a pairing needed to be recorded separately. Both S1 and S2 expressed uncertainty about the distinction between the codomain and the range. S4 and S5 omitted the range from their written responses, but their reasons were not confirmed through interviews. These findings suggest that understanding set definitions and membership provides an important foundation for representing relations. This pattern is consistent with Afgani et al. (2022) and Pratiwi (2022), who examined students' misconceptions about relations and functions, and with Kefi et al. (2021), who used CRI to identify students' difficulties with relations.

Representational Accuracy in Expressing Relations

Errors in ordered-pair notation suggest that some students had not distinguished between set names and set elements. Mathematically, an ordered pair consists of a domain element as its first component and a codomain element as its second; the set names merely identify the sets to which these elements belong. Using set labels as components of ordered pairs can therefore obscure the meaning of the relation being represented. This finding is consistent with Iffa et al. (2022), who examined mathematical representation abilities in relations and functions, and Arfany & Faradiba (2022), who investigated the characteristics of students' conceptual errors in the same topic. Nevertheless, the reasons underlying the notation were not confirmed through interviews for all participants in this study. The findings concerning S1, S3, and S5 should therefore be interpreted as errors evident in their written responses rather than definitive conclusions about their thinking processes.

Procedural Difficulties in Calculating Function Values

Difficulties at the carrying-out-the-plan stage indicate that calculating function values and consistently applying a formula need to be considered separately. S3 correctly obtained $f(2) = 7$through substitution but then performed an additional operation by dividing the result by 2. S6 correctly calculated some function values but did not apply the complete expression $f(x) = 2x + 3$when completing the table. S5 explained the substitution procedure correctly but showed insufficient care and expressed uncertainty about the recorded results. These differences indicate that incorrect or incomplete responses do not necessarily stem from misconceptions. Instead, some may reflect difficulties in executing procedures, maintaining consistency in formula application, or checking results. Teachers should therefore ask students to explain the reasoning behind each step rather than assess only their final answers. A systematic examination of errors is also important for locating students' difficulties, as emphasized by Lestari & Rosdiana (2021).

Problem-Solving Strategies Across Polya's Stages

The analysis using Polya's stages indicates that difficulties at one stage can affect subsequent stages. At the understanding-the-problem stage, most participants identified the domain, but uncertainty about the distinction between the codomain and the range was evident for S1 and S2 and suggested by the written responses of S4 and S5. At the devising-a-plan stage, students' use of ordered pairs to represent relations was not always grounded in an understanding of the elements of the two sets. For example, S2 wrote $(A, 1)$, $(A, 2)$, and so forth, treating the set name as the first component, whereas S6 correctly wrote $\{(1, 2), (2, 4), (3, 6), (4, 8)\}$. At the carrying-out-the-plan stage, S4 and S5 correctly explained direct substitution into $f(x) = 2x + 3$. In contrast, S3's written response showed an additional division by 2 after obtaining $f(2) = 7$, while S6 entered $1x$, $2x$, $3x$, and $4x$ in the table. At the looking-back stage, S1 relied on the existence of a corresponding element for each domain element. S3 initially used the same approach before recognizing that the domain element 2 was paired with two different elements. In the written responses, only S6 concluded that the relation was not a function.

This pattern indicates that looking back requires more than checking whether every domain element is connected to an element in the codomain; it also requires verifying that each

Kognitif: Jurnal Riset HOTS Pendidikan Matematika
https://doi.org/10.51574/kognitif.v6i4.6073 Volume 6, Issue 4, October - December 2026, pp. 1828 –1847

domain element has exactly one corresponding element. These findings relate to the studies by Nisa & Asmarani (2023) and Idris & Imami (2024), who analyzed mathematical problem solving using Polya's stages. The present study extends this work by examining how the identified conceptual and representational difficulties relate to students' strategies at each stage. When examined in relation to performance on the CRI-based diagnostic test, the findings did not follow a strictly linear pattern. The high-performing participants, S1 and S6, demonstrated different strategies. S6 represented ordered pairs correctly and concluded at the looking-back stage that the relation was not a function, whereas S1 repeatedly listed set elements and checked only for the existence of corresponding elements. The medium-performing participants, S2 and S3, had difficulties with ordered-pair notation, and S3 also had difficulty continuing the calculation appropriately. Nevertheless, S3 explained the concept of range and the defining conditions of a function orally. The low-performing participants, S4 and S5, correctly applied substitution at the carrying-out-the-plan stage, although S5 showed insufficient care. Both omitted the range and classified the relation as a function at the looking-back stage. These patterns suggest that diagnostic test performance does not necessarily correspond to success at every stage of Polya's framework: difficulties in checking the uniqueness requirement occurred across all three performance categories, while procedural success was also demonstrated by low-performing students.

The findings also emphasize the importance of interpreting CRI ratings alongside answer accuracy and students' explanations. Incorrect answers accompanied by high confidence may indicate misconceptions, whereas incorrect answers accompanied by low confidence are more appropriately interpreted as a lack of conceptual understanding. Correct answers accompanied by low confidence do not necessarily demonstrate secure understanding. These interpretive principles follow the use of CRI introduced by Hasan et al. (1999). In this study, interviews helped clarify some incomplete written responses. For example, S3 explained the concept of range orally despite omitting it from the written response and revised the explanation of the defining conditions of a function following probing questions. The interview with S1 revealed the reasoning behind repeatedly listing set elements. However, interviews did not cover all participants or all stages, so their clarifying role was partial. Moreover, responses elicited through probing cannot be equated with students' ability to check their answers independently. Misconceptions should therefore be identified cautiously rather than inferred from a single response or data source.

Limitations

This study has several limitations. *First*, the CRI instrument was developed by the researcher and had not undergone formal validation by experts or mathematics teachers. The resulting classifications should therefore be interpreted as diagnostic indications based on the combination of answer accuracy and confidence, rather than as a sole basis for identifying misconceptions. *Second*, diagnostic test data were collected from 28 students in a single class, while the in-depth analysis involved only six purposively selected participants from one school. The findings are therefore not intended to be generalized to a broader population. *Third*, the data were collected during a single data collection period within one semester. Consequently, the study does not capture changes in students' understanding over time. *Fourth*, interviews were conducted with only five participants and focused on selected aspects of their responses. S6 was not interviewed, and not all of Polya's stages were clarified through interviews. Some interpretations were therefore based solely on CRI results and written responses. Future research should systematically validate the instruments, involve participants from multiple

classes or schools, collect data at multiple time points, and conduct interviews covering all participants and relevant problem-solving stages.

Implications for Mathematics Instruction

The findings suggest that instruction on relations and functions should provide opportunities for students to construct conceptual meaning and connect multiple representations. Teachers can use examples and non-examples to compare relations that are functions with those that are not, asking students to justify their classifications. Arrow diagrams, ordered pairs, and tables should be explicitly connected so that students understand that these representations express the same relationship. To strengthen understanding of the range, students can be asked to compare all elements of the codomain with those that are actually paired with domain elements. During problem-solving activities, teachers can establish a step-by-step checking routine: verify that every domain element has a corresponding element, check that each has exactly one corresponding element, and then compare the conclusion with the definition of a function. CRI-based diagnostic assessment can provide an initial source of information, but it should be supplemented with constructed-response problems and probing questions to develop a more comprehensive understanding of students' conceptual understanding and strategies. These implications are consistent with previous research addressing misconceptions, representations, and reasoning in relations and functions (Arfany & Faradiba, 2022; Iffa et al., 2022; Saputri et al., 2023).

Conclusion

The findings indicate that eighth-grade students at SMP Negeri 2 Parepare exhibited various misconceptions and mathematical difficulties when solving problems involving relations and functions. Conceptual findings primarily concerned their understanding of the domain, codomain, range, and the conditions under which a relation qualifies as a function. Representational errors involved the use of ordered-pair notation and the listing of set elements. Procedural difficulties included calculating function values, consistently applying formulas when completing tables, and checking the accuracy of calculations. The CRI results showed that the number of responses indicating misconceptions varied across participants; their interpretation therefore requires consideration of response completeness and students' confidence levels.

The analysis using Polya's stages revealed variations in students' problem-solving strategies across understanding the problem, devising a plan, carrying out the plan, and looking back. At the understanding-the-problem stage, difficulties primarily involved identifying the range and understanding the defining conditions of a function. At the devising-a-plan stage, some students used ordered-pair representations incorrectly. At the carrying-out-the-plan stage, difficulties involved applying substitution procedures and completing tables consistently. At the looking-back stage, some students did not verify that each domain element had exactly one corresponding element. Conceptual errors concerning the defining conditions of a function and the range appeared to affect understanding the problem and looking back, whereas representational errors affected devising a plan. Nevertheless, S3 revised the explanation of the defining conditions of a function after receiving probing questions.

These findings highlight the need to identify misconceptions by considering answer accuracy, confidence levels measured through CRI, written work, and students' interview

Kognitif: Jurnal Riset HOTS Pendidikan Matematika
https://doi.org/10.51574/kognitif.v6i4.6073 Volume 6, Issue 4, October - December 2026, pp. 1828 –1847

explanations where clarification was obtained. Teachers should strengthen conceptual understanding, develop students' ability to move between representations, and foster the habit of verifying answers against mathematical definitions and properties. The findings should be interpreted in light of the study's limitations: the CRI instrument had not undergone formal validation; the in-depth analysis involved six participants from a single class; data were collected during a single data collection period; and interviews covered only five participants and selected problem-solving stages. Future research should involve a broader participant sample, validate the instruments, and adopt a more comprehensive interview protocol.

Declarations

Conflict of Interest	:	The authors declare that there are no conflicts of interest.
Auhor Contributions	:	M.A.M.: Writing – review & editing. M.B.: Conceptualization, Writing – review & editing. A.H.: Conceptualization, Investigation, Formal analysis, Writing – original draft, Writing – review & editing. D.Y.B.: Conceptualization, Investigation, Formal analysis, Writing – original draft. A.S.Y.: Conceptualization, Investigation, Formal analysis. The author approved the final version of the manuscript.
Data Availability Statement	:	The data supporting the findings of this study are available from the corresponding author, A.H., upon reasonable request
AI Statement	:	The authors declare that generative AI tools were used to improve the clarity, grammar, and readability of the English language.

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Authors Biography

	**Marwati Abd Malik** serves as a university-based teaching practicum supervisor (DPL) in the Mathematics Teacher Professional Education (PPG) Program at Universitas Muhammadiyah Parepare. Her areas of expertise include mathematics education and the academic supervision of students in the PPG program. ✉ marwati.pare63@gmail.com
	**Mas'ud B** is a lecturer in the Mathematics Teacher Professional Education (PPG) Program at Universitas Muhammadiyah Parepare. His areas of expertise include professional practice and the academic supervision of students in the PPG program. ✉ umpar.masud@umpar.ac.id

Kognitif: Jurnal Riset HOTS Pendidikan Matematika
https://doi.org/10.51574/kognitif.v6i4.6073 Volume 6, Issue 4, October - December 2026, pp. 1828 –1847

	**Arsy Handayani** is a student in the Mathematics Teacher Professional Education (PPG) Program at Universitas Muhammadiyah Parepare. She is currently undertaking her teaching practicum (PPL) at SMP Negeri 2 Parepare. Her research interests include mathematical misconceptions and students' mathematical problem-solving strategies. ✉ arsyhandayani44@gmail.com
	**Dewi Yuniarti Bayu** is a student in the Mathematics Teacher Professional Education (PPG) Program at Universitas Muhammadiyah Parepare. Her research interests focus on the analysis of mathematics learning difficulties and diagnostic assessment. ✉ dewiyuniartibayu29@gmail.com
	**Abdul Salam Yusuf** is a mentor teacher at SMP Negeri 2 Parepare, where he guides students in the Teacher Professional Education (PPG) Program during their teaching practicum (PPL). His area of expertise is mathematics instruction at the junior high school level. ✉ abdulyusuf63@guru.smp.belajar.id