KOGNITIF JURNAL RISET HOTS PENDIDIKAN MATEMATIKA

Volume 6 Issue 4 2026, October – December pp. 1749 – 1764
e-ISSN : 2776-9704
p-ISSN: 2776-9984
https://doi.org/10.51574/kognitif.v6i4.5984

Comparing Students' Critical Thinking Skills in Arithmetic Sequences: Self-Directed Learning versus Contextual Teaching and Learning

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Citation: Siregar, Y. E., & Lubis, R. S. (2026). Comparing Students' Critical Thinking Skills in Arithmetic Sequences: Self-Directed Learning versus Contextual Teaching and Learning. *Kognitif: Jurnal Riset HOTS Pendidikan Matematika*, 6(4), 1749–1764. https://doi.org/10.51574/kognitif.v6i4.5984

ARTICLE INFO

ARTICLE HISTORY:

Received Sep 04, 2026
Accepted Oct 26, 2026
Published Online Oct 28, 2026

KEYWORDS:

Arithmetic Sequences and Series
Contextual Teaching and Learning
Mathematical Critical Thinking
Self-Directed Learning

ABSTRACT

Developing mathematical critical thinking requires instructional approaches that support students in interpreting problems, selecting strategies, and drawing justified conclusions. Comparing approaches with different emphases on contextual support and learner control can inform instructional decisions in arithmetic sequences and series. This study compared the mathematical critical thinking performance of Grade 10 students taught through Self-Directed Learning (SDL) and Contextual Teaching and Learning (CTL). A quantitative, quasi-experimental study employed a posttest-only nonequivalent comparison-group design involving 60 students from two intact classes at a secondary school in Medan, Indonesia. Each class comprised 30 students and received four instructional meetings. Performance was assessed using two constructed-response problems addressing problem identification, analysis, problem solving, and conclusion drawing. Data were examined using descriptive statistics and an exploratory individual-level Welch comparison. The CTL class obtained a higher mean posttest score ($M = 85.47, SD = 7.35$) than the SDL class ($M = 71.13, SD = 10.00$), with an observed difference of 14.33 points. The exploratory comparison yielded $t(53.26) = 6.33$, $p < .001$, and a standardised observed difference of Hedges' $g = 1.61$. However, the absence of a pretest and the inclusion of only one class per condition preclude conclusions about improvement or causal superiority. The study contributes a topic-specific comparison of two active instructional approaches, highlighting the need to examine how contextual support and learner control are organised in relation to mathematical task demands.

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Introduction

Critical thinking is a central goal of mathematics education, requiring students to move beyond obtaining correct answers to examining information, evaluating alternative strategies, and justifying their conclusions (Henderson & Hurley, 2013; Kuntze et al., 2017; Lehmann,

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2023). Mathematical proficiency therefore involves not only procedural accuracy but also the ability to determine whether a solution is logically sound and adequately supported (Cromley et al., 2017). Research on critical thinking in mathematics education highlights the importance of designing learning experiences that cultivate these capabilities (Lestari et al., 2021; Yasin et al., 2019). Instructional approaches should therefore provide students with meaningful opportunities to analyse problems, articulate their reasoning, and critically review their solutions.

Arithmetic sequences and series offer a productive context for investigating mathematical critical thinking. Solving problems in this domain requires students to identify patterns, establish relationships among terms, select appropriate strategies, and interpret results in relation to the conditions of a problem (Dibbs, 2019; Muhtadi et al., 2017; Susanta et al., 2023). However, the ability to recall and apply formulas does not necessarily imply an understanding of their underlying relationships or conditions of use. Research on adaptive reasoning in arithmetic sequences and series has identified difficulties in justifying solutions and explaining the conceptual connections between sequences and series, particularly when students encounter nonroutine problems (Finesilver, 2022; Pittalis et al., 2018). These difficulties highlight the need for instruction that develops reasoning alongside procedural competence.

Preliminary evidence from an interview with a Grade 10 mathematics teacher at SMA Budi Satrya Medan suggests similar instructional challenges. The teacher reported that students were generally more comfortable solving exercises that closely resembled previously demonstrated examples but struggled when the structure or demands of a problem changed. Students were also reported to participate infrequently in asking questions and explaining their mathematical ideas. Although these observations do not establish students' levels of critical thinking, they indicate a need for learning opportunities that encourage independent strategy selection, reasoned justification, and the evaluation of mathematical solutions.

Self-Directed Learning (SDL) offers one approach to addressing this need by engaging students actively in directing their learning, while retaining appropriate instructional support. In their systematic review of SDL in science, technology, engineering, and mathematics education, Bishara (2021), Manaud & Aggabao (2024), and Suhaimee et al. (2025) identified both its educational potential and the challenges associated with its implementation, particularly when demands for learner autonomy intersect with complex subject matter. In the context of arithmetic sequences and series, SDL can provide opportunities for students to identify gaps in their understanding, select learning resources, explore solution strategies, and evaluate their work (Istiqlal & Himmah, 2019; Pokhrel et al., 2024; Sumantri & Satriani, 2016). These activities offer a plausible pathway through which SDL may support mathematical critical thinking, although this relationship requires empirical examination within specific instructional settings.

Contextual Teaching and Learning (CTL) provides another relevant approach by connecting mathematical concepts with situations that students can interpret meaningfully. A systematic review by Mentari & Syarifuddin (2020) and Sevinc & Lesh (2022) characterises CTL as an approach that links new knowledge to prior experience and situates learning within authentic contexts. For arithmetic sequences and series, such contexts may include seating arrangements, savings that increase by a fixed amount, or the cumulative number of objects in a growing pattern (Lee & Lee, 2021). Rather than serving merely as a narrative setting, a context can provide a basis for identifying relevant information, constructing mathematical models, and assessing the reasonableness of solutions.

Although SDL and CTL both support active participation, they emphasise different aspects of the learning process. SDL foregrounds students' responsibility for directing and evaluating their learning (Phoodee, 2023; Sekano et al., 2026; Turan & Koç, 2018), whereas CTL foregrounds the construction of meaning through connections between mathematical concepts and their applications (Nabila & Widjajanti, 2020; Taher et al., 2019). These contrasting emphases provide a theoretical rationale for comparing students' critical thinking skills across the two approaches. Nevertheless, SDL and CTL are not mutually exclusive: self-directed activities can incorporate contextual problems, and contextual instruction can include opportunities for independent learning. A meaningful comparison therefore depends on clearly specifying the defining features of each approach and ensuring that these features are reflected in classroom implementation.

Existing research supports the educational relevance of both approaches, but its implications must be considered in relation to the populations, mathematical content, and outcomes investigated. The systematic review of SDL in STEM education addresses a range of learning outcomes rather than directly comparing SDL with CTL in relation to critical thinking in arithmetic sequences and series. Similarly, a meta-analysis of CTL in mathematics education examined mathematical understanding, while a study focusing specifically on mathematical critical thinking through CTL involved prospective primary school teachers. The literature reviewed here therefore provides a foundation for the present study but does not directly resolve how Grade 10 students' mathematical critical thinking skills differ when the same content is taught through SDL and CTL. Accordingly, this study compares the mathematical critical thinking skills of Grade 10 students taught through SDL and CTL in arithmetic sequences and series at SMA Budi Satrya Medan. It addresses the following research question: To what extent do students' mathematical critical thinking skills differ between the SDL and CTL groups? By providing comparative evidence within a clearly defined mathematical domain and school setting, the study seeks to inform instructional decisions based on the competencies these approaches are intended to develop, rather than on their broadly stated educational benefits.

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Method

Research Approach and Design

This quantitative study employed a quasi-experimental, posttest-only nonequivalent comparison-group design. Two intact Grade 10 classes received different instructional approaches: Self-Directed Learning (SDL) and Contextual Teaching and Learning (CTL). Both classes studied arithmetic sequences and series and completed a mathematical critical thinking test after the instructional programme. The documented procedures do not describe random assignment of students to the instructional conditions. The grouping variable was instructional approach, with two categories: SDL and CTL. The outcome was mathematical critical thinking, operationalised through four assessed components: problem identification, analysis, problem solving, and conclusion drawing. The total test score represented overall performance within the scope of the instrument, while component scores provided a more detailed description of student performance (In Table 1).

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Table 1. Research design

Group	Class	Instruction	Pretest	Posttest
First comparison group	X-A	SDL	Not administered	Mathematical critical thinking test
Second comparison group	X-B	CTL	Not administered	Mathematical critical thinking test

Because no pretest was administered, the study examined differences in postinstruction performance rather than improvement from baseline. Moreover, only one class represented each instructional condition. Consequently, differences associated with instruction could not be disentangled from pre-existing or contextual differences between the classes. Findings should therefore be interpreted as observed differences between the two participating classes, rather than as generalisable causal effects of the instructional approaches.

Settings and Participants

The study was conducted at SMA Swasta Budi Satrya Medan, North Sumatra, Indonesia, for 4 months. The reported accessible population comprised 60 Grade 10 students in two intact classes. All students in these classes were included through a census approach, described in the original report as total sampling. Class X-A comprised 30 students and received SDL instruction, while Class X-B comprised 30 students and received CTL instruction. Existing class membership was retained. Inclusion of all students in the accessible population describes participant recruitment, not random assignment to instructional conditions. Participant characteristics, including age range, gender distribution, previous exposure to the topic, and prior mathematics attainment, should be reported where available. The number completing the posttest and the number included in the analysis must be stated separately if absence or incomplete data occurred. The source document reports 29 observations for one class in the variance-comparison table, whereas the t-test table and score lists contain 30 students per class. This discrepancy must be resolved before the final sample is reported.

Insructional Procedures

Each class participated in four lessons supported by lesson plans and student worksheets adapted to its instructional condition. The implementation account specifies two instructional periods per lesson, whereas parts of the teaching module specify four 45-minute periods. Actual instructional exposure must therefore be established from the timetable and implementation records and reported as 90 minutes per lesson. In the SDL condition, students were given opportunities to seek information, develop their understanding, and discuss their learning with peers and the teacher. The teaching materials describe the use of books or internet resources, group activities, presentations, and reflection. Accordingly, SDL in this study should be described as supported self-directed activity incorporating teacher guidance and peer interaction, rather than as individual learning without assistance. In the CTL condition, learning involved problems connected to real-life situations. Students discussed these problems in groups, related contextual information to arithmetic sequences or series, and developed solutions. The teacher provided guidance and facilitated discussion of mathematical concepts and student work in Table 2.

Table 2. Documented characteristics of the instructional conditions

Component	SDL	CTL
Instructional emphasis	Information seeking and student-directed development of understanding with teacher support	Conceptual understanding through connections between mathematics and contextual problems
Reported student activities	Seeking information, completing worksheets, discussing ideas, and presenting findings	Interpreting contextual problems, discussing ideas, completing worksheets, and presenting solutions
Resources or starting points	Worksheets, books, internet resources, and teacher guidance	Real-life problems and worksheets
Interaction	Independent and group activities	Group activities and whole-class discussion
Teacher role	Guiding information seeking, monitoring activities, and discussing student understanding	Supporting interpretation of contexts, facilitating discussion, and reviewing solutions
Content	Arithmetic sequences and series	Arithmetic sequences and series
Outcome assessment	Posttest	Posttest

Both conditions included discussion and worksheets, and contextual problems also appeared in parts of the SDL materials. The distinction between conditions therefore cannot rest solely on the presence of group work or real-life contexts. The final account must clarify the degree of student control over learning in SDL and the role of context in concept development in CTL, using evidence from actual implementation. If fidelity was not systematically assessed, it should not be presented as an established feature of the study.

Instruments

The primary instrument was a constructed-response test assessing mathematical critical thinking in arithmetic sequences and series. It comprised two main problems, each with four response sections, yielding eight scored sections mapped to four assessed components. This structure is consistent with the test blueprint and the appended test; the reference to ten items in the original analysis section should therefore be corrected. The first problem concerned salary changes involving a fixed increment, while the second concerned monthly savings with a fixed increase. Students were asked to identify relevant information, select a mathematical representation or formula, show their solution procedures, and state the resulting answer. Mathematical critical thinking was operationally restricted to performance on the four assessed components; the total score was not treated as a comprehensive measure of all possible dimensions of critical thinking in Table 3.

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Table 3. Test blueprint and operational definitions

Component	Operational definition	Evidence examined in responses	Sections	Maximum per problem
Problem identification	Recognising the given information and the demands of the problem	Relevant information, known quantities, and the quantity to be determined	1a, 2a	10
Analysis	Decomposing the problem and identifying the required mathematical relationships	Identification of the first term, common difference, number of terms, and appropriate formula	1b, 2b	10
Problem solving	Connecting information and concepts to develop a solution	Substitution, calculation steps, and a solution appropriate to the problem	1c, 2c	15
Conclusion drawing	Stating a result consistent with the question and its context	A conclusion consistent with the working and the requested quantity	1d, 2d	15

Data Collection

The principal outcome data were obtained from the posttest administered after instruction in Table 4 and Figure 1. Lesson plans and worksheets supported the instructional conditions, while test scores provided the primary basis for comparing mathematical critical thinking performance. The preliminary teacher interview informed the description of the instructional context; it did not replace direct assessment of student performance.

Table 4. Data collection stages

Stage	Activity	Records or data
Preparation	Developing lesson plans, worksheets, test questions, and rubric	Initial instructional materials and instrument
Expert review	Reviewing the instrument with a lecturer and teacher	Review records and instrument revisions
Pilot testing	Administering the instrument to the pilot participants	Responses and instrument-analysis data
SDL instruction	Four lessons in Class X-A	Worksheet responses and implementation records
CTL instruction	Four lessons in Class X-B	Worksheet responses and implementation records
Outcome assessment	Administering the postinstruction test	Posttest scripts
Scoring	Applying the rubric to student responses	Section scores and total scores
Data preparation	Reconciling scores, scripts, and participant records	Verified analytical dataset

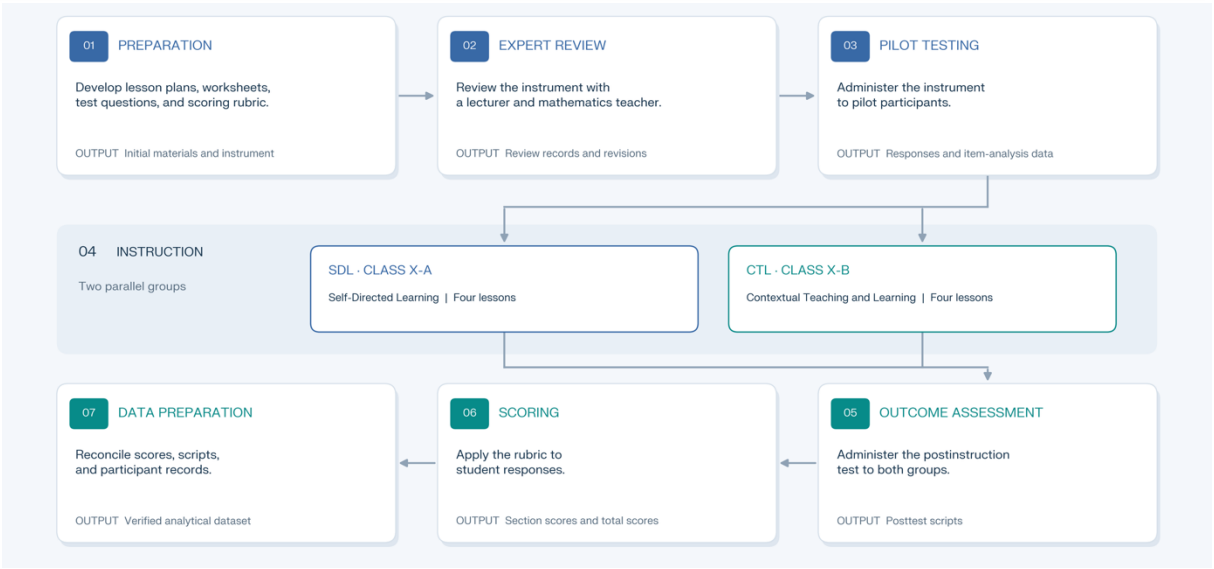


Figure 1. Research Procedures Comparing SDL and CTL in Mathematical Critical Thinking

Analysis

Before analysis, scores should be reconciled with the test scripts and rubric to ensure that all values fall within the permitted ranges. Checks should cover participant counts, completeness, agreement between section scores and total scores, and data-entry accuracy. The analytical dataset should include participant codes, class, instructional condition, eight section scores, four component scores, and the total score. Student names are not required in the analytical dataset. Performance should be summarised for each group using the analytical sample size, mean, standard deviation, median, minimum, and maximum. Component scores should be converted to a common 0–100 scale before comparing performance profiles. The total score should be the primary outcome. Component-level analyses should be described as descriptive or exploratory unless separate inferential tests were specified and supported by adequate measurement evidence.

The comparison should use a two-sided hypothesis: the null hypothesis specifies equal mean posttest performance in the SDL and CTL groups, while the alternative specifies a difference in either direction. A consistent direction should be defined for the observed mean difference: $\Delta = mean(CTL) - mean(SDL)$. Positive values indicate higher observed mean scores in CTL, and negative values indicate higher observed mean scores in SDL. The original report used a two-sample t-test assuming equal variances. If an individual-level inferential comparison is retained in the reanalysis, Welch's t-test may be considered because it does not require equal variances. Its use need not depend on a preliminary significant variance-homogeneity test. However, neither version of the t-test resolves the one-class-per-condition design limitation. If individual-level p values or confidence intervals are reported, they must be explicitly identified as exploratory and conditional on a strong independence assumption that may not hold within classes. The data cannot empirically separate class differences from instructional-condition differences. Statistical significance should not be interpreted as demonstrating an isolated causal effect of instruction. In addition to the raw mean difference, a standardised difference such as Hedges' g may be reported after the data have been verified. This quantity describes the observed score difference; it does not establish causality.

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Confidence intervals based on individual-level independence require the same qualification concerning classroom structure. Change scores, normalised gain, and pretest-adjusted ANCOVA are not supported by the documented procedures because no baseline critical thinking assessment is reported. All conclusions should therefore concern postinstruction performance rather than improvement.

Research Findings

Implementation of the Instructional Approaches

The implementation account reported four instructional meetings in each condition, with 30 students in Class X-A receiving Self-Directed Learning (SDL) and 30 students in Class X-B receiving Contextual Teaching and Learning (CTL). Instruction was delivered by the school mathematics teacher and supported by lesson plans and student worksheets. The report described both independent and collaborative activities in the SDL condition, whereas the CTL condition used real-life problems as starting points for group discussion and mathematical problem solving. Table 5 summarises the reported implementation.

Table 5. Reported implementation of SDL and CTL

Feature	SDL condition	CTL condition
Participating class	X-A	X-B
Reported class size	30 students	30 students
Instructional meetings	Four	Four
Instructional provider	School mathematics teacher	School mathematics teacher
Reported learning activities	Independent information seeking, group discussion, and review of understanding with the teacher	Interpretation of real-life problems, group discussion, and development of mathematical solutions
Teacher support	Encouragement, guidance, and discussion of students' developing understanding	Contextual problem prompts and support for discussion
Supporting records	Classroom documentation and appended worksheet-response materials	Classroom documentation and appended worksheet-response materials
Quantified implementation fidelity	Not established from the available report	Not established from the available report

In the SDL class, students were given time to seek information about arithmetic sequences and series before discussing their understanding with peers and the teacher. Group work was also incorporated, indicating that the approach combined opportunities for independent information seeking with collaborative discussion. The implementation narrative described the teacher as providing encouragement and subsequently reviewing the mathematical content in relation to the understanding students had developed. The researcher's discussion reported that students were not yet accustomed to SDL and continued to require teacher direction when developing their understanding of the topic. This account identifies a practical implementation challenge: opportunities for independent activity did not eliminate the need for instructional guidance. However, no systematic observation scores or quantified records of the support

required were presented. The account therefore represents a descriptive implementation observation rather than a measured difference in learner autonomy.

In the CTL class, real-life problems were used to initiate learning activities. Students worked in groups to discuss the problems and develop mathematical solutions. The reported implementation thus foregrounded contextual problem interpretation and collaborative problem solving, rather than beginning primarily with independent information seeking. The CTL module organised the intended activities around relating, experiencing, applying, cooperating, and transferring. These stages clarify the planned instructional structure, but the available implementation evidence does not establish the extent to which each stage occurred in every meeting. The findings therefore support a descriptive account of the learning activities rather than a numerical assessment of adherence to the CTL framework.

Available Posttest Data and Descriptive Statistics

The appended posttest score lists contained 60 records, with 30 students in each instructional group. Descriptive statistics were recalculated from these individual scores rather than copied from the inconsistent summary tables in the original report. These calculations verify the numerical summaries of the listed scores but do not constitute an independent audit of the underlying scoring. Table 6 presents the recalculated descriptive statistics.

Table 6. Descriptive statistics for mathematical critical thinking posttest scores

Group	n	Mean	SD	Median	Q1	Q3	Min.	Max.
SDL	30	71.13	10.00	73.00	66.00	75.00	50	97
CTL	30	85.47	7.35	85.00	81.00	89.00	77	100

The CTL class obtained a higher mean posttest score (M = 85.47, SD = 7.35) than the SDL class (M = 71.13, SD = 10.00), with an observed difference of 14.33 points. Median scores showed the same pattern. Scores were also less dispersed in the CTL class, as indicated by its smaller standard deviation and narrower observed range. Supplementary distributional checks were conducted on the transcribed scores. The Shapiro–Wilk test indicated a departure from normality in the CTL group, $W = 0.874$, $p = 0.002$. The corresponding test for the SDL group was not statistically significant, $W = 0.954$, $p = 0.219$. The median-centred Levene test did not provide sufficient evidence of unequal variances, $F(1, 58) = 1.503$, $p = 0.225$. These findings do not establish independence of observations or remove the study's classroom-level design limitation (See, in Table 7).

Table 7. Supplementary distributional checks

Test	Group or comparison	Statistic	p
Shapiro–Wilk	SDL	0.954	0.219
Shapiro–Wilk	CTL	0.874	0.002
Median-centred Levene test	SDL and CTL	1.503	0.225

Exploratory Comparison of Posttest Scores

An exploratory individual-level Welch comparison yielded a mean difference of 14.33 points in favour of the CTL class, $t(53.26) = 6.33$, $p < .001$. Under the individual-independence assumption, the 95% confidence interval for the mean difference extended from 9.79 to 18.88

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points. The corresponding standardised observed difference was Hedges' g = 1.61. Table 8 presents exploratory comparision of mathematics critical thinking

Table 8. Exploratory comparison of mathematical critical thinking posttest scores

Comparison	Mean difference	95% CI	Welch t	df	p	Hedges g
CTL relative to SDL	14.33	[9.79, 18.88]	6.33	53.26	< .001	1.61

This comparison is supplementary and exploratory. The departure from normality in the CTL group qualifies the model-based inference, while the use of only one intact class per condition is the more fundamental design constraint. Neither Welch's test nor its confidence interval distinguishes instructional differences from class-specific differences. Accordingly, the results describe higher observed posttest performance in the participating CTL class; they do not establish that CTL caused the difference. The absence of a pretest also precludes conclusions about improvement from baseline.

Discussion

Interpreting the Observed Performance Difference

The comparison revealed higher mathematical critical thinking posttest scores in the CTL class than in the SDL class. The recalculated means were 85.47 and 71.13, respectively, corresponding to an observed difference of 14.33 points. This finding is meaningful within the participating sample, but its interpretation must remain aligned with the design. Because no baseline assessment was administered, the difference represents postinstruction performance rather than improvement from an established starting point. Similarly, the standardised difference describes the separation between the observed score distributions; it does not independently establish an instructional effect. The distinction between a numerical difference and an instructional explanation is particularly important here. Each approach was implemented in only one intact class, so instructional condition coincided with class membership. Differences in prior attainment, classroom relationships, familiarity with contextual problems, or other unmeasured characteristics could therefore contribute to the outcome. The exploratory individual-level statistical comparison does not separate these possibilities. Methodological work on one-cluster-per-condition designs highlights precisely this confounding of group characteristics and intervention conditions. The result should consequently be understood as a bounded comparison of two instructional implementations in arithmetic sequences and series. Its value is not that it establishes a universal ranking of CTL and SDL, but that it identifies a performance pattern that requires explanation in relation to task demands, instructional support, and assessment design (Manaud & Aggabao, 2024; Suhaimee et al., 2025; Sumantri & Satriani, 2016). This interpretation preserves the empirical finding without attributing more explanatory power to the design than it can support.

Contextual Interpretation and the Demands of Arithmetic Sequences and Series

A plausible explanation for the observed pattern concerns the relationship between contextual interpretation and mathematical representation. Problems involving arithmetic sequences and series require students to determine what changes by a constant amount, identify

the starting quantity, establish the relevant number of terms, and distinguish an individual term from an accumulated sum (Kertil et al., 2019). A student may recall the correct formulas yet select an inappropriate one if the meaning of the quantities or the question is misunderstood. The implementation narrative described real-life problems as starting points in the CTL condition. This arrangement may have helped organise attention around the relationship between a situation and its mathematical structure. For example, interpreting the amount saved in a particular month differs from determining cumulative savings over several months. Discussing the meaning of these quantities can potentially make the distinction between a term and a sum more explicit. This is an explanation of a possible instructional opportunity, not a claim that the present study directly measured that process.

The literature study by Yasin et al. (2019) and Zanden et al. (2020) addresses mathematical critical thinking through CTL and is relevant to the contextual rationale for this interpretation. Its scope, however, is not a direct comparison showing that SDL is ineffective. The original discussion's use of this source as evidence against SDL should therefore be corrected. A literature-based rationale can establish the relevance of contextual instruction, but it cannot identify the mechanism responsible for the present class difference (Phoodee, 2023; Sekano et al., 2026). The available data do not show whether students in the CTL class interpreted contexts more accurately, chose formulas more appropriately, or justified their choices more fully. Those possibilities require separate evidence from student responses or classroom discourse. The contextual explanation is therefore best presented as a theoretically plausible account that helps specify what future analysis should examine.

Self-Direction, Learner Readiness, and the Organisation of Support

The lower mean in the SDL class should not be interpreted as evidence that self-directed learning lacks educational value. The implementation account reported that students were unfamiliar with the approach and continued to require teacher direction. This observation points to a possible implementation challenge: giving students responsibility for seeking information does not guarantee that they can select, integrate, and evaluate that information in relation to a mathematical problem. Information access and mathematical understanding are not equivalent. A student may locate a formula without recognising its conditions of use, or reproduce a worked example without understanding how a new problem differs. In this sense, the relevant issue is not simply how much independence was offered, but what decisions students were prepared to make and what support accompanied those decisions (Dagdag & Calimag, 2023; Evin Gencel & Saracaloğlu, 2018; Hasibuan et al., 2026). The study did not measure these processes directly, so they cannot be presented as established explanations for the observed scores.

Hasibuan et al. (2026) reviewed empirical research on SDL in STEM teaching and learning and identified both its potential and implementation challenges. This broader evidence supports examining the conditions under which self-direction is introduced rather than assuming that autonomy either succeeds or fails uniformly. It also cautions against generalising from one classroom implementation to the effectiveness of SDL. The two conditions may have differed in where support was located within the learning sequence. In the SDL account, students sought information before reviewing their understanding with the teacher. In the CTL account, the contextual problem provided an organising focus for discussion. This difference could influence what students attended to and when misunderstandings were addressed. Nevertheless, the report did not quantify the amount, timing, or quality of teacher assistance, so the comparison cannot isolate support as a mediator. A useful interpretation is therefore that learner control and instructional support should be considered together. Independence is not

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defined simply by the absence of teacher explanation. Future implementations could examine whether students are able to identify learning needs, select relevant resources, explain strategy choices, and evaluate conclusions, while documenting how teacher assistance is adjusted to those capabilities.

Positioning the Findings Within Previous Research

The relationship to earlier research should be framed through the comparability of instructional conditions, participants, mathematical content, and outcomes. The CTL literature study by Ramda et al. (2023), Mentari & Syarifuddin (2020), and Nur (2024) address relevant educational approaches, but neither constitutes a direct replication of this classroom comparison. They support the relevance of the research question rather than settling which condition should produce the higher score. This distinction prevents a selective interpretation of the literature. Sources supporting CTL should not be treated as showing that SDL is inferior unless they compare the two approaches. Likewise, research describing positive SDL outcomes cannot establish that the present implementation was successful on outcomes that were not measured. Differences across studies may reflect variation in task complexity, students' experience, duration, instructional support, or the construct assessed, but those possibilities must be examined rather than presumed. The study's contribution is therefore specific: it documents an observed posttest difference between two intact classes studying the same mathematical domain under different instructional arrangements. That contribution becomes more informative when paired with a transparent account of implementation and measurement boundaries. The key question is not merely which label was associated with the higher mean, but which opportunities to interpret, explain, and review mathematical relationships were available to students. Other citations in the original discussion, including Lestari et al. (2021) and Velani & Retnawati (2020) , should be reintegrated only after the original sources have been checked. Their findings should be distinguished by subject area, intervention, outcome, and research design. They have not been used here to support unverified comparisons.

Implications for Instructional Design

The findings suggest a practical focus on the coordination of task meaning, learner responsibility, and teacher support. For arithmetic sequences and series, discussion can be organised around identifying the changing quantity, explaining the role of the common difference, and distinguishing a term from a cumulative sum. These are proposed instructional priorities arising from the demands of the topic and the observed comparison, not component-level gains demonstrated by the present data. When contextual tasks are used, the context should support mathematical decision making rather than function only as a familiar story. Students can be asked to explain why a particular formula applies, compare alternative interpretations, and check whether the units and magnitude of an answer fit the question. Such activities would also make the intended meaning of critical thinking more explicit in instruction and assessment. For SDL, possible refinements include prompts for identifying what needs to be learned, criteria for evaluating the relevance of resources, checkpoints for explaining strategy choices, and opportunities to review conclusions. These supports should be presented as design proposals requiring investigation, not as interventions whose effectiveness was demonstrated in this study. Their purpose would be to support the quality of student decisions rather than merely increase the amount of independent work. An integrated design is also conceivable: contextual

problems could provide a common starting point while students exercise increasing control over resource selection, strategy development, and self-evaluation. The present comparison did not test such an integration. It nevertheless illustrates why contextual support and self-direction need not be treated as competing principles. Implications for curriculum or policy should remain modest.

Conclusion

This study found higher mathematical critical thinking posttest scores in the class taught through Contextual Teaching and Learning than in the class taught through Self-Directed Learning in arithmetic sequences and series. The exploratory analysis indicated a difference in posttest performance between the two groups. However, the absence of a pretest and the inclusion of only one intact class per condition prevent this difference from being interpreted as improvement from baseline or as evidence of the causal superiority of CTL over SDL. The implementation account highlighted distinct instructional emphases. In the CTL class, contextual problems served as starting points for discussion and mathematical problem solving. In the SDL class, students sought information before reviewing their understanding with peers and the teacher. Nevertheless, the available evidence did not establish which instructional processes accounted for the score difference or whether student engagement differed between the conditions. The study's contribution is therefore a context-specific comparison of postinstruction performance within the participating classes and the scope of the assessment. These findings underscore the importance of considering the alignment among mathematical tasks, learner readiness, and instructional support. Future research should involve multiple classes per condition, assess baseline performance, use a broader range of mathematical tasks, and systematically document classroom implementation. Such evidence would provide a stronger basis for understanding how contextual support and learner control relate to students' mathematical critical thinking.

Declarations

Conflict of Interest : The authors declare that there are no conflicts of interest.

Auhor Contributions : Y.E.S.: Conceptualization, data curation, formal analysis, funding acquisition, investigation, methodology, resources, supervision, visualization, and writing original draft.
R.S.: Data curation, investigation, project administration, validation, visualization, and writing review and editing.
All authors have read and approved the final version of the manuscript.

Data Availability Statement : The data supporting the findings of this study are available from the corresponding author, R.S., upon reasonable request

AI Statement : The authors state that generative AI tools were used solely to improve the clarity, grammar, and readability of the English language in the manuscript.

Kognitif: Jurnal Riset HOTS Pendidikan Matematika
https://doi.org/10.51574/kognitif.v6i4.5984 Volume 6, Issue 4, October - December 2026, pp. 1749 –1764

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