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How to cite: Azisah, N., Ma'rup, M., Halim, S. N. H., & Ilhamsyah, I. (2026). Students' Understanding of Algebraic Concepts and High Self-Efficacy: A Case Study in Junior High School. *Kognitif: Jurnal Riset HOTS Pendidikan Matematika*, 6(3), 1295–1312. <https://doi.org/10.51574/kognitif.v6i3.4948>

To link to this article: <https://doi.org/10.51574/kognitif.v6i3.4948>



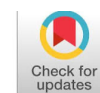
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Students' Understanding of Algebraic Concepts and High Self-Efficacy: A Case Study in Junior High School

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Article Info

Article history:

Received Apr 04, 2026

Accepted Aug 08, 2026

Published Online Aug 28, 2026

Keywords:

Algebraic Thinking

Conceptual Understanding

Junior High School

Mathematics

Qualitative Case Study

Self-Efficacy

ABSTRACT

Research on self-efficacy in mathematics learning has thus far focused primarily on students with low self-efficacy or those experiencing learning difficulties; consequently, the conceptual understanding profiles of students with high self-efficacy particularly regarding algebraic concepts at the junior high school level remain largely unexplored. This study aims to describe the mathematical conceptual understanding of students categorized as having high self-efficacy in algebraic expressions. The study employed a qualitative case study approach with a single seventh-grade student who had the highest self-efficacy score as the sole subject, selected purposively based on the results of a validated self-efficacy questionnaire and teacher recommendations. Data were collected through a conceptual understanding test and in-depth interviews, analyzed using Miles' procedures for data condensation, data presentation, and data verification, with data validity ensured through methodological triangulation. The results showed that the subject consistently met all seven indicators of mathematical concept understanding, as reflected in his ability to reconstruct the meaning of concepts in his own words, logically analyze the necessary conditions of an algebraic expression, translate real-world contexts into algebraic models, and explain the reasoning behind each step in solving algebraic subtraction problems. The results showed that the subject consistently met all seven indicators of mathematical concept understanding, as reflected in his ability to reconstruct the meaning of concepts in his own words, logically analyze the necessary conditions of an algebraic expression, translate real-world contexts into algebraic models, and explain the reasoning behind each step in solving algebraic subtraction problems. These findings suggest that high self-efficacy serves as a cognitive resource that supports coherent and self-regulated conceptual reasoning, providing an initial profile that can serve as a basis for comparative studies across levels of self-efficacy.



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Introduction

Students' understanding of mathematical concepts remains a serious challenge in mathematics education at various levels. Many students merely memorize formulas without understanding the underlying concepts, causing them to struggle when faced with problems presented in forms different from the examples they have studied. Mathematics is a discipline that plays a crucial role in developing logical, critical, and structured thinking (Marni & Pasaribu, 2021), and conceptual understanding is one of its core skills. According to Ansari (Fajar et al., 2019), a focus on conceptual understanding helps students gain in-depth knowledge through learning experiences and connect one concept to another, which ultimately contributes to improved learning outcomes (Sihombing et al., 2021).

One psychological factor believed to influence conceptual understanding is self-efficacy that is, an individual's belief in their ability to complete a specific task. Self-efficacy has also been reported to be associated with academic achievement in mathematics and to influence how students act, strive, and persevere when faced with difficulties (Halim et al., 2023). Students with high self-efficacy tend to be more willing to explore various problem-solving strategies and are less likely to give up when faced with unfamiliar problems. A study by Rahmi et al., (2020) found that self-efficacy has a significant effect on the ability to understand mathematical concepts, meanwhile Nurani et al., (2021) reported that students with high self-efficacy tend to master all indicators of conceptual understanding, an achievement not always found among students with low self-efficacy (Wirdania et al., 2024). Perseverance and self-confidence among students with high self-efficacy (Fitri, 2017) theoretically supports the development of a more relational and meaningful conceptual understanding.

However, these studies are mostly quantitative in nature and focus on correlational relationships between variables; thus, they do not yet explain how the cognitive process of conceptual understanding occurs in students with high self-efficacy. Other studies have measured conceptual understanding based on its seven indicators (Giawa et al., 2022) or exploring students' conceptualization processes regarding algebra and functions, but have not specifically linked these processes to students' self-efficacy profiles. Thus, there remains a limited body of qualitative research that delves deeply into the profiles and cognitive processes underlying mathematical concept understanding among students with high self-efficacy, particularly regarding algebraic expressions in junior high school (SMP).

Preliminary observations conducted through interviews with mathematics teachers and an analysis of seventh-grade students' work at a public junior high school revealed striking variations in conceptual understanding among students: most students struggled to apply concepts to problems different from the examples provided, while a small minority were able to provide adequate conceptual explanations and demonstrated greater initiative and self-confidence a pattern indicating varying levels of self-efficacy. Teachers also emphasized that students who are able to explain the meaning of concepts rather than merely memorizing formulas are generally those who are more willing to try and more confident in their own abilities. These preliminary findings underscore the urgency of conducting an in-depth study of how students with high self-efficacy build their conceptual understanding, so that the findings can serve as a guide for designing instruction that fosters self-efficacy in other students.

Algebraic expressions specifically, their elements and the operations of addition and subtraction were chosen as the context for this study because they form an important foundation for advanced topics such as linear equations, functions, and systems of equations, and are also among the topics that most frequently pose difficulties for middle school students. These difficulties generally stem from students' tendency to memorize procedures without

understanding the underlying concepts (Fajar et al., 2019). Qualitative studies linking conceptual understanding of algebraic expressions to self-efficacy are still very limited; therefore, this study offers a novel approach to exploring in depth how students with high self-efficacy develop conceptual understanding of this material. Based on the above discussion, this study aims to describe the mathematical conceptual understanding of students categorized as having high self-efficacy (hereinafter referred to as SET subjects) regarding algebraic expressions and operations in 7th-grade junior high school.

Algebra Problems for Students

Algebraic expressions are consistently reported as one of the topics that seventh-grade middle school students find difficult to understand. Isfayani (2023) found that most students had difficulty understanding concepts, principles, and verbal language in algebraic form, with difficulties in conceptual understanding accounting for the largest proportion. These difficulties generally stemmed from a weak grasp of the meanings of variables, coefficients, and constants, as well as errors when performing operations on like terms (Herawati & Kadarisma, 2021). An international study by Jupri et al., (2014) also emphasizes that the transition from arithmetic thinking to algebraic thinking is a crucial point that gives rise to various misconceptions among Indonesian students, particularly in interpreting letters as variables and in generalizing number patterns. These difficulties stem largely from students' tendency to memorize procedures without understanding the underlying concepts (Fajar et al., 2019), as a result, students easily lose their way when faced with variations in question formats. Understanding how students' cognitive processes construct conceptual understanding of algebraic expressions—particularly among students who possess high self-efficacy as a psychological resource—is crucial for identifying learning strategies that can potentially be replicated by other students who are still struggling.

Self-Efficacy in Learning

Self-efficacy in mathematics learning refers to students' belief in their own ability to understand the material and solve mathematical problems (Bandura, 1997). These beliefs can influence students' actions, efforts, resilience, and commitment to achieving their learning goals (Halim et al., 2023). Bandura (2012) breaks down self-efficacy into three dimensions: magnitude (the level of difficulty of tasks one believes they can complete), strength (the strength of one's belief in their ability despite facing obstacles), and generality (the breadth of that belief across various task contexts). These three dimensions are interrelated in shaping students' learning behaviors, including persistence, strategy selection, and resilience in the face of failure when encountering challenging problems. A number of studies indicate that self-efficacy plays a significant role in the quality of students' mathematical understanding and problem-solving. Utami & Wutsqa, (2017) found a positive relationship between self-efficacy and mathematical problem-solving ability, in which students with high self-efficacy were more persistent in trying various strategies when they encountered a dead end. Pratiwi & Imami, (2022) It also confirms that students with high self-efficacy tend to be more confident and better able to solve math problems than students with low self-efficacy. Thus, self-efficacy is not merely a secondary psychological factor, but also shapes the way students approach, process, and interpret the mathematical concepts they are learning.

Method

Type of Research

This study employs a descriptive qualitative approach with a single-case study design (Creswell, J. W., & Poth, 2018). This design was chosen because the purpose of the study was to explore in depth the thought processes and characteristics of mathematical concept understanding in a single student with high self-efficacy as a case study subject examined intensively, rather than to statistically test the relationships among variables. A case study allows researchers to obtain a comprehensive picture of the case under study through various complementary data sources.

Subject

The study was conducted among seventh-grade students in the Algebra program at Mappakasunggu Public Junior High School 2 during the 2025/2026 school year. The research subjects were selected using purposive sampling from among 19 students who completed the self-efficacy questionnaire. The selection of subjects was based on specific considerations, namely students with the highest self-efficacy scores in the high self-efficacy category, as well as recommendations from subject teachers who assessed the students' suitability for the characteristics required in the study (Sugiyono, 2019). Based on these criteria, one subject with the initials MAI hereinafter referred to as SET was selected, with a self-efficacy score of 90.2. A single subject was selected for this study to obtain in-depth information about the phenomenon under investigation. This is consistent with the nature of qualitative research, which emphasizes depth of information over generalizations about the population. In qualitative research, samples or data sources may be selected purposively based on research considerations and objectives (Sugiyono, 2019). Thus, the selection of a single subject in this study was not intended to represent the population quantitatively, but rather to obtain an in-depth understanding in line with the focus of the study.

Instruments

Three instruments were used in this study: a self-efficacy questionnaire, a mathematical concept comprehension test, and a semi-structured interview guide. The self-efficacy questionnaire was adapted from Hendriana (2017). An instrument specifically developed to measure students' mathematical self-efficacy, with content validity tested through expert judgment and reliability estimated using Cronbach's alpha. The conceptual understanding test consisted of three open-ended questions on algebraic terms and operations (addition and subtraction), each designed to cover more than one indicator of conceptual understanding simultaneously (details of the question-indicator mapping are presented in Table 1). A semi-structured interview guide was used to explore the subjects' thought processes in a flexible yet focused manner, centering on the seven indicators of conceptual understanding that served as the framework for this study (presented in full in Table 1). All three instruments were validated by two expert validators, and the additional conceptual understanding test was administered to students outside the study sample to assess its readability.

Table 1. Summary and Sample Instrumen Items

Instruments	Functions	Example item
self-efficacy Questionnaire	Measuring and categorizing students' levels of mathematical self-efficacy (1–4 Likert scale), adapted from Hendriana (2017)	"I am confident that I can solve difficult math problems if I try my hardest" (strength dimension)
Conceptual Understanding Test (Questions 1–3)	Three essay questions on algebraic expressions and operations; discount of Rp2,000. Question 1 assesses learning for the total cost of Ani's and Budi's purchases! outcome 1.1, Question 2 covers Determine and simplify the difference between the total learning outcomes 1.2, 1.3, and 1.5, costs of Ani's and Budi's purchases! (c) If one notebook and Question 3 covers learning costs Rp3,000 and one pen costs Rp2,000, calculate the outcomes 1.4, 1.6, and 1.7	"Ani bought 4 notebooks and 3 pens, while Budi bought 2 notebooks and 5 pens. Each purchase received a discount of Rp2,000. (a) Write the algebraic expressions for the total cost of Ani's and Budi's purchases! (b) Determine and simplify the difference between the total learning outcomes 1.2, 1.3, and 1.5, costs of Ani's and Budi's purchases! (c) If one notebook and one pen costs Rp3,000 and one pen costs Rp2,000, calculate the difference in the amounts paid by Ani and Budi!" (Problem 3)
Interview Guidelines	Semi-structured interviews to explore the subjects' thought processes regarding the seven coefficients, and constants mean in algebraic indicators of conceptual expressions." (Indicator 1.1) understanding	"Try to explain in your own words what variables, processes regarding the seven coefficients, and constants mean in algebraic indicators of conceptual expressions." (Indicator 1.1)

The seven indicators of conceptual understanding that serve as the framework for this study are: restating concepts, classifying objects according to specific properties, providing examples and non-examples, presenting concepts in various representations, developing necessary and sufficient conditions, selecting specific procedures or operations, and applying concepts to problem-solving.

Collections

Data collection was conducted over three sessions between October and November 2025. The first session (30 minutes) involved administering a self-efficacy questionnaire to the entire class. The second session (60 minutes) involved administering a conceptual understanding test to selected subjects in the classroom under conducive conditions. The third session (30–45 minutes) consisted of in-depth interviews with the subjects to confirm their test answers and explore their thought processes; these were recorded using a voice recorder and supplemented with field notes to ensure the completeness of the data.

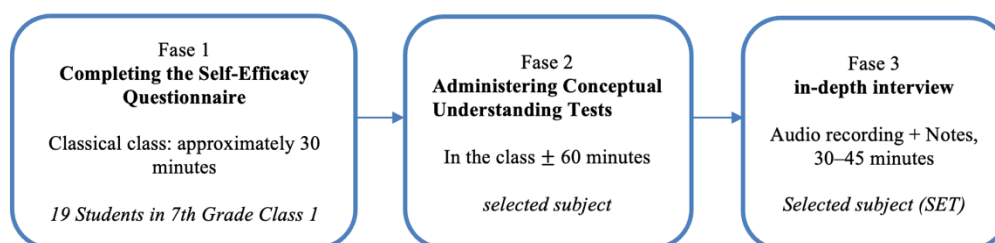


Figure 1. Research Procedures

Analysis

The data were analyzed using the [Miles et al., \(2014\)](#) model, which consists of three stages: (1) data condensation, which involves selecting, simplifying, and transforming raw data

from tests and interviews into a more meaningful form; (2) data presentation, which involves systematically organizing the information into triangulation tables and descriptive narratives; and (3) drawing conclusions and verification, which involves determining the fulfillment of each indicator based on the consistency between test data and interview data. The coding process involves labeling each data unit: the code “P” followed by the question number for the interviewer’s questions (e.g., P1, P2.A, P2.B) and the code “SET” followed by the same question number for the subject’s responses (e.g., SET1, SET2.A, SET2.B). These codes serve only as turn-taking markers, whereas the actual units of analysis being coded are the forms of cognitive behavior in each response—such as reconstructing the meaning of a concept, classifying objects, or applying procedures—which are then mapped to one of the seven indicators of conceptual understanding. Each indicator is assessed based on two sources of evidence: written responses on the test sheet and verbal statements during the interview. An indicator is considered met if both data sources show consistent results. Data validity is achieved through methodological triangulation—that is, comparing test data and interview data from the same subject—in accordance with the criteria for credibility in qualitative research.

Research Findings

The study began with the administration of a self-efficacy questionnaire, followed by a 3-question test of conceptual understanding of algebraic expressions (elements and operations of algebraic expressions), and concluded with interviews. The results of the self-efficacy questionnaire are presented in [Tables 2](#) and [Table 3](#).

Table 2. Results of the Student Self-Efficacy

No.	Student ID	Self-Efficacy survey results	category
1.	DHH	78,6	Medium
2.	NA	69,6	Medium
3.	RPI	77,7	Medium
4.	SRA	84,8	High
5.	NES	69,6	Medium
6.	MRAR	68,8	Medium
7.	NPA	77,7	Medium
8.	ISL	75,0	Medium
9.	DA	68,8	Medium
10.	SBA	78,6	Medium
11.	MAI	90,2	High
12.	MSF	70,5	Medium
13.	AM	67,0	Medium
14.	ASS	63,4	Low
15.	SY	65,2	Medium
16.	KRA	70,5	Medium
17.	KAP	62,5	Low
18.	SFAZ	67,0	Medium
19.	ANH	70,5	Medium

Based on the [Table 2](#), there are 2 students in the high self-efficacy category, 15 students in the moderate category, and 2 students in the low category. This summary was derived from grouping students based on self-efficacy, as reflected in the scores obtained from the self-efficacy questionnaire. Of the two students in the high category—SRA (84.8) and MAI (90.2)—this study selected MAI as the sole case study subject because he had the highest score and was recommended by the subject teacher as the strongest representative of the high self-efficacy category in his class; he will henceforth be referred to as subject SET.

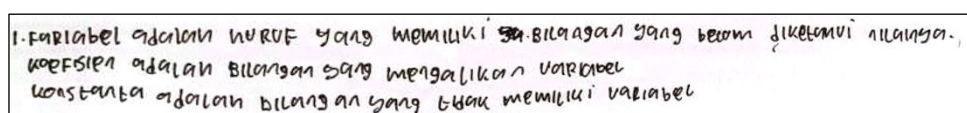
Table 3. Research Subjects

No	Student's initials	Student ID	Categories of Student Self-Efficacy
1	MAI	SET	High

This section explains the understanding of mathematical concepts among students in the SET (High Self-Efficacy) group when solving problems involving algebraic terms and operations; this explanation is based on seven indicators of mathematical concept understanding.

Rearticulating a Concept

SET states that a variable is a letter representing a number whose value is unknown. A coefficient is defined as a number that multiplies a variable, while a constant is a number that does not involve a variable. Based on the results of the conceptual understanding test and interviews, SET participants were able to restate a concept, as evidenced by their ability to write down and explain the definitions of variables, coefficients, and constants accurately and coherently. The following are the results of the SET's mathematical concept comprehension test in solving algebraic operations.



1. Variabel adalah huruf yang memiliki sa. Bilangan yang belum diketahui nilainya.
 koefisien adalah bilangan yang mengalikan variabel
 konstanta adalah bilangan yang tidak memiliki variabel

Figure 2. SET test results

Based on Figure 2, the SET is capable of restating concepts. This is evident in the SET's responses, which demonstrate an accurate understanding of what variables, coefficients, and constants are. The following are the results of the SET's interview regarding question number 1 for the "restating concepts" indicator.

- Code : Description
- P1 : What is the question asking in that problem?
- SET1 : Explain the meanings of variables, coefficients, and constants in algebraic expressions in
 ir own words.
- P1 : Okay, so please explain the definitions of variables, coefficients, and constants!
- SET1 : So, a variable is a letter that represents a number whose value is unknown. A coefficient
 : number that multiplies a variable. A constant, on the other hand, is a number that does
 : contain a variable.

Classifying Objects According to Specific Characteristics in Accordance with Their Concepts

SET grouped the following algebraic expressions: (i) $4x + 3y - 2x + 5 + y$, (iii) $2a + 5 - a + 4$, dan (v) $3p - 2q + p - 4$. Meanwhile, (ii) $7 + 3 - 2 + 5$, (iv) $6 - 3 + 2 - 1$, dan (vi) $9 - 5 + 2$. Based on the test results and interviews, the SET subjects met criterion 1.2, which is classifying objects according to specific properties in accordance with their concepts. The following are the results of the SET mathematical concept comprehension test in solving algebraic expression problems.

2.A. Bentuk Aljabar: -i: $4x+3y-2x+5+y$ -ii: $2a+5-a+4$ -v: $3p-2q+p-4$	Bukan bentuk Aljabar: -ii: $7+3-2+5$ -iv: $6-3+2-1$ -vi: $9-5+2$
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Figure 3. SET2.A test results

Based on Figure 3, SET is capable of classifying objects according to specific properties in accordance with their concepts. This is evident in the results, which show that SET was able to correctly group several shapes into algebraic and non-algebraic categories. The following are the results of the SET interview regarding question number 2 in Part A for the indicator “classifying objects according to specific properties in accordance with their concepts.”

Code	: Description
P2.A	: After reading the question, what information can be inferred from question number 2, part A?
SET2.A	: The question asks you to categorize which ones are algebraic expressions and which are not, along with your reasoning.
P2.A	: Okay, so out of that set of expressions, which ones are algebraic expressions and which ones aren't?
SET2.A	: examples of algebraic expressions include (i) $4x + 3y - 2x + 5 + y$; (iii) $2a + 5 - a + 4$; (v) $3p - 2q + p - 4$
P2.A	: So, which one is not an algebraic expression?
SET2.A	: Parts (ii) $7 + 3 - 2 + 5$; (iv) $6 - 3 + 2 - 1$; and (vi) $9 - 5 + 2$

Giving Examples and non-examples of a concept

SET is able to accurately provide examples of variables, coefficients, and constants in an algebraic expression. In addition, SET is also able to identify elements that are not variables, coefficients, or constants. Based on the interview results, SET can explain the differences between examples and non-examples clearly and fluently. Based on the results of the concept comprehension test and the interview, the SET subject was able to provide examples and non-examples of a concept, as evidenced by the ability to correctly identify and name examples of variables, coefficients, and constants. The following are the results of the SET concept comprehension test in solving algebraic expression problems.

B: $4x+3y-2x+5+y$ variabel: x, y koefisien: 4 pada $4x$, 3 pada $3y$, -2 pada $-2x$ konstanta: 5
iii: $2a+5-a+4$ variabel: a koefisien: 2 pada $2a$ konstanta: $5, 4$
v: $3p-2q+p-4$ variabel: p, q koefisien: 3 pada $3p$, -2 pada $-2q$ konstanta: -4

Figure 4. SET2.B test results

Based on Figure 4, SET is capable of providing examples and non-examples of a concept. This can be seen in the results, which show that SET is already able to provide examples of variables, coefficients, and constants. The following are the results of the SET interview on question number 2, part B, regarding the indicator of providing examples and non-examples of a concept.

Code	: Description
P2.B	: What the question is asking?
SET2.B	: Please identify the variables, coefficients, and constants.
P2.B	: As mentioned earlier, these are considered algebraic expressions. From the expressions identified as algebraic, can you identify which ones are variables, coefficients, and constants?
SET2.B	: So, in the form (i) $4x + 3y - 2x + 5 + y$, the variables are: x, y ; the coefficients are: 4 for $4x$, 3 for $3y$, and -2 for $-2x$; the constant is: 5. In the form (iii) $2a + 5 - a + 4$, the variables are: a ; the coefficient is: 2 for $2a$; the constant is: 5, 4. Meanwhile, in the form (v) $3p - 2q + p - 4$, the variables are: p, q ; the coefficients are: 3 for $3p$, and -2 for $2q$; the constant is: -4

Presenting Concepts in Various Forms of Mathematical Representation

SET lets the price of a notebook be represented by the variable x and the price of a pen by the variable y . Next, SET formulates an algebraic expression for the total cost of Ani and Budi's groceries. Based on the test results and interviews, the SET subjects were able to present concepts in various forms of mathematical representation. The following are the results of the SET's mathematical concept comprehension test in solving algebraic operation problems.

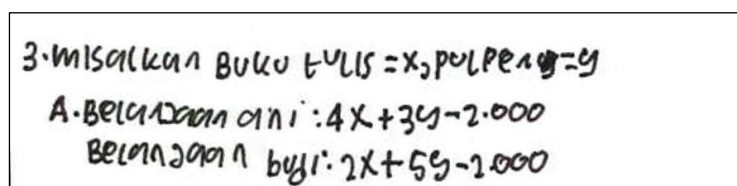


Figure 5. SET3.A test results

Based on Figure 5, SET is capable of presenting concepts in various forms of mathematical representation. This is evident in the results, which show that SET is able to accurately interpret the given word problem and convert it into a mathematical model. The following are the results of the SET interview regarding question number 3, part A, for the indicator “presenting concepts in various forms of mathematical representation.”

Code	: Description
P3.A	: After reviewing the question, what is known and what is being asked in the question?
SET3.A	: As far as we know, Ani bought 4 notebooks and 3 pens, while Budi bought 2 notebooks and 5 pens, and each purchase came with a discount of Rp Rp 2.000
P3.A	: Is the information provided in the question sufficient to answer it?
SET3.A	: enough
P3.A	: Okay, so what else is being asked in the question?
SET3.A	: The question is, write the algebraic expression that represents the total cost of Ani and Budi's groceries.
P3.A	: What steps were taken to resolve the issue?
SET3.A	: I'll represent the price of a notebook with a variable—I'll use the variable x for the notebook and y for the pen. Since the problem states that Ani bought 4 notebooks and 3 pens, while Budi bought 2 notebooks and 5 pens, and we are asked to write the algebraic expressions for the total cost of Ani's purchases and the total cost of Budi's purchases, I wrote $4x + 3y - 2.000$. and $2x + 5y - 2.000$.
P3.A	: Try to summarize the answer
SET3.A	: So I conclude that Ani's grocery bill is $4x + 3y - 2.000$. And Budi's grocery bill is $2x + 5y - 2.000$.

Developing Necessary or Sufficient Conditions for a Concept

SET explains that an expression is considered algebraic because it contains variables, such as x, y, a , and so on. Conversely, expressions that do not contain variables are not considered algebraic. Based on the results of the tests and interviews, SET participants were able to accurately identify the necessary and sufficient conditions for a concept. The following are the results of the SET concept comprehension test in solving problems involving algebraic expressions.

ALCISANJUA: KARENA BEKUT ALJABAR MEMILIKI VARIABEL, SEPERTI x, y, a , DAN SEBAGAIKUNYA
ALJABAR: KARENA TIDAK ADA VARIABEL YANG MENYIKUJANYA,

Figure 6. SET2.A test results

Based on Figure 6, SET is considered capable of developing the necessary and sufficient conditions of a concept in solving problems involving algebraic expressions and operations, as he is able to explain the reasoning behind classifying elements as algebraic expressions or non-algebraic expressions. Judging by his answers, SET was able to solve the problems correctly. The following are the results of SET's interview regarding question number 2 in Part A, which assessed his ability to develop the necessary and sufficient conditions of a concept.

Code	: Description
P2.A	: Why is that set an algebraic form and not an algebraic form?
SET2.A	: The sets (i, iii, v) are algebraic expressions because they contain variables. Examples include x, y, a , and so on.
P2.A	: Then, which set do the sets (ii, iv, vi) belong to?
SET2.A	: Meanwhile, (ii, iv, vi) are not algebraic expressions because they are not followed by any variables.

Using, Applying, and Selecting Specific Procedures or Operations

SET writes the difference between Ani's and Budi's grocery bills in algebraic form. Next, SET expands the parentheses, multiplies the negative signs, and groups like terms until the final result is obtained. Based on the test results and interviews, SET is able to use, apply, and select specific procedures or operations to solve problems correctly. The following are the results of SET's mathematical concept comprehension test in solving algebraic operation problems.

$$\begin{aligned}
 &B \cdot (4x + 3y) - 7.000 - (2x + 8y) - 7.000 \\
 &= 4x + 3y - 7.000 - 2x + 8y + 7.000 \\
 &= 4x - 2x + 3y - 5y - 7.000 + 7.000 \\
 &= 2x - 2y
 \end{aligned}$$

Figure 7. SET3.B test results

Based on Figure 7, SET is capable of using, applying, and selecting specific procedures or operations—including subtraction and grouping like terms—to solve problems accurately. The following are the results of the SET interview regarding question number 3 in Part B for the indicator “using, applying, and selecting specific procedures or operations.”

Code	: Description
P3.B	: After reading the question, what do we know from it?

- SET3.B : From what I understand about the problem, we're asked to write the algebraic expression for the difference between Ani's and Budi's grocery bills and then simplify it.
- P3.B : Could you explain what procedure or method was used to solve that problem?
- SET3.B : Since the question asks for the difference in the cost of Ani's and Budi's groceries, I used subtraction, which is $((4x + 3y) - 2000) - ((2x + 5y) - 2000)$
- P3.B : Okay. What's the next step? Please explain each step of the problem-solving process!
- SET3.B : Next, I expanded the parentheses to get $4x + 3y - 2000$, Then I multiplied the minus sign by Budi's grocery bill, resulting in $-2x - 5y + 2000$. After that, I grouped like terms: $4x - 2x + 3y - 5y - 2000 + 2000$ so the result is $2x - 2y$

Applying Concepts or Algorithms to Problem Solving

SET substituted the values of x and y into the equation. The calculation showed that the difference between the amounts paid by Ani and Budi was Rp 2.000. During the interview, SET correctly summarized the answer and expressed confidence in it. The following are the results of SET's mathematical concept comprehension test in solving algebraic operation problems.

$2x - 2y$
 $x = 3.000$
 $y = 2.000$
 $= 2(3.000) - 2(2.000)$
 $= 6.000 - 4.000$
 $= 2.000$
 Jadi selisih uang yang dibayar Ani dan Budi yaitu Rp. 2.000

Figure 8. SET3.C test results

Based on Figure 8, SET is capable of applying concepts or algorithms to problem-solving. This is evident from the results, which show that SET was able to solve the problem using the correct steps and arrive at the correct answer. The following are the results of the SET interview regarding question number 3, part C, for the indicator “applying concepts or algorithms to problem-solving.”

- Code : Description
- P3.C : After reading the question, what is known and what is being asked in the question?
- SET3.C : We know that a notebook costs Rp 3,000 and a pen costs Rp 2,000. The question is: what is the difference between the amounts Ani and Budi paid?
- P3.C : Can this problem be solved without solving the problem in part b?
- SET3.C : No
- P3.C : Okay. So what's the first step to solving that problem?
- SET3.C : So, I entered the final answer from part b into part c. Then I let $x = \text{notebook}$ and $y = \text{pen}$, My first step was to set up the equation $2x - 2y$, because the problem states that one notebook costs Rp. 3000 and one pen costs Rp 2000, So, I substituted $x = 3000$, and $y = 2000$. so $2(3000) - 2(2000)$ then I multiplied them to get $6000 - 4000$ resulting in 2000
- P3.C : Try to summarize the answers you've found
- SET3.C : What I can conclude is that the difference between the amounts paid by Ani and Budi is Rp 2000
- P3.C : Are you sure about the answer?
- SET3.C : Yeah, sis, I'm sure

Based on the test results and interviews, the SET subject was able to apply concepts or algorithms to problem-solving, as evidenced by the ability to plug values into an algebraic model and then calculate the difference accurately and with confidence. Based on the work samples and interview excerpts above, the SET subject meets the indicator of applying concepts or algorithms to problem-solving. After analyzing the written responses and summarizing the interview data from the SET subject, the data was entered into a table to facilitate drawing conclusions based on the analysis, as follows.

Table 4. Achievement and Triangulation of Concept Understanding Indicators for SET Subject

Conceptual Understanding Indicators	Test results	interview	interpretation
Reararticulating a Concept	SET should rephrase the definitions of variables, coefficients, and constants accurately and in their own words.	SET is able to explain verbally what variables, coefficients, and constants are in a logical sequence without referring to a book.	SET has reconstructed the meaning of the concept, rather than merely memorizing its definition. This demonstrates a relational understanding, not merely an instrumental one.
Classifying Objects According to Specific Characteristics in Accordance with Their Concepts	SET correctly classifies algebraic and non-algebraic expressions.	SET can explain the rationale for grouping based on the presence of variables.	SET understands the conceptual criteria underlying the classification, rather than simply guessing. This ability reflects a comprehensive understanding of the conceptual structure of algebraic forms.
Providing examples and counterexamples of a concept	SET accurately lists examples and non-examples of variables, coefficients, and constants.	The SET lists and provides examples and non-examples of each algebraic element.	SET is able to apply conceptual understanding to concrete examples and distinguish them from non-examples, which is an indication that pemahaman konsep yang kuat.
Presenting concepts in various forms of mathematical representation	SET accurately converts word problems into algebraic models, representing the prices of notebooks and pens with different variables.	SET can explain the process of translating from real-world contexts into mathematical language.	SET demonstrates the ability to translate representations from verbal language into mathematical symbols. This ability is important as a bridge between conceptual and procedural understanding.
Developing the necessary or sufficient conditions for a concept	SET stipulates that the existence of variables is a necessary condition for a form to be called algebraic.	SET explains that without variables, a form cannot be classified as an algebraic form.	SET is capable of analyzing the logical relationships between parts of a concept and determining its boundaries. This demonstrates an

Conceptual Understanding Indicators	Test results	interview	interpretation
			understanding that goes beyond rote memorization and leads to conceptual reasoning.
Using, applying, and selecting specific procedures or operations	SET selects subtraction operations and groups like terms using a systematic and correct approach.	SET outlines the steps involved in the resolution process and the reasons for selecting that procedure.	SET did not merely follow the procedures mechanically, but understood the reasoning behind the choice of procedures. Although successful, SET hesitated at times when sequencing the steps, indicating that procedural flexibility could still be improved.
Applying concepts or algorithms to problem-solving	SET correctly outlines the steps to solve the problem and arrives at the correct final answer.	SET explains the steps taken, correctly summarizes the answer, and demonstrates strong confidence in the answer.	SET is able to integrate concepts and procedures to comprehensively solve contextual problems. High self-efficacy is reflected in perseverance and the courage to make decisions when solving problems.

Based on Table 4, the triangulation results between the test data and the SET subjects' interviews showed consistency across all seven indicators; thus, it can be concluded that SET successfully met all seven indicators of mathematical concept understanding. Although SET met all seven indicators, this achievement was not attained instantly or without obstacles. For example, SET initially showed hesitation in sequencing the steps of the problem-solving process before finally determining the correct order of subtraction operations, as evident from the pauses and repeated explanations in the interview excerpt SET3.B. This pattern indicates that SET's thinking strategy is not always linear: he occasionally tries, evaluates, and corrects his own approach before arriving at the final answer. The dominant strategy used by SET was to first represent the real-world situation using algebraic symbols, then verify the result through value substitution—a pattern that consistently emerged in the solutions to questions 3A through 3C. The difficulties identified were not related to conceptual understanding but rather to procedural aspects—specifically, the sequence of steps when an operation involved more than one stage. It is precisely at these moments of uncertainty that the role of self-efficacy becomes evident: SET students did not stop or ask for help but independently retraced their steps until they were confident in the results, as indicated in the P3 interviews. B. These findings reinforce the argument that conceptual understanding among students with high self-efficacy is dynamic, accompanied by a process of self-regulation in the face of uncertainty, rather than merely a flat achievement devoid of challenges.

Discussion

The findings of this study indicate that students with the highest self-efficacy scores consistently met all seven indicators of mathematical concept understanding in the topic of algebraic expressions and operations, as reflected in their ability to reconstruct the meaning of concepts in their own words, logically analyze the necessary conditions of an algebraic

expression, and solve algebraic operations while explaining their reasoning at each step. These findings are consistent with previous quantitative studies that reported a positive association between self-efficacy and conceptual understanding. (Rahmi et al., 2020 ; Nurani et al., 2021), however, a question that these studies have not yet answered is how this cognitive process occurs at the individual level.

The most basic explanation can be found in self-efficacy theory Bandura, (1997), which asserts that an individual's belief in their own abilities influences their choice of cognitive activities, the amount of effort they exert, and their persistence in the face of obstacles. This mechanism is consistent with the SET model, which involves not merely attempting to answer questions but actively constructing meaning and maintaining cognitive engagement. This mechanism of self-belief then manifests itself in concrete cognitive processes that can be explained through constructivism Piaget, J., & Inhelder (1972): Meaningful understanding is formed through the assimilation and accommodation of cognitive schemas, and students with high self-efficacy tend to engage more actively in accommodation because they do not shy away from challenges. This is evident in the ability of students with high self-efficacy to reconstruct definitions of variables, coefficients, and constants in their own words, as well as to recognize the essential nature of a concept through examples—and not through non-examples—which reflects an active accommodation process rather than passive information storage.

This accommodation process, in turn, results in a level of understanding that can be distinguished using the framework Skemp (1976) between instrumental understanding (knowing the rules without understanding the reasons behind them) and relational understanding (knowing the rules along with their connections to other concepts). Students with high self-efficacy demonstrate relational understanding, for example, when analyzing that the presence of variables is a necessary condition for a form to be categorized as algebraic, and when explaining that the distribution of negative signs in subtraction is not merely a memorized procedure but is understood structurally. The ability to translate representations demonstrated by students with high self-efficacy when converting a shopping context into an algebraic model can be further explained through cognitive load theory (Sweller, 1988): The more organized cognitive schemas of students with high self-efficacy reduce the cognitive load involved in processing complex problems, allowing them to shift their attention from mere computation to an understanding of the underlying concepts.

Compared to previous research, these findings expand our current understanding in three ways. Rahmi et al., (2020) while previous studies have quantitatively confirmed the relationship between self-efficacy and conceptual understanding, they have not revealed the underlying mechanisms; this study fills that gap by explaining the cognitive processes behind that relationship. Nurani et al., (2021) By examining the profile of conceptual understanding across self-efficacy categories without delving into any specific category, this study provides a level of depth not achieved by cross-category studies. Wirdania et al., (2024) examining the topic of linear equations, while this study focuses on the algebraic form of the critical transition point from arithmetic thinking to algebraic thinking (Kieran, 1992), which presents a qualitatively different cognitive challenge—not merely a different topic.

The finding that specifically distinguishes this study is the discovery of nuances in the process of students with high self-efficacy: although they successfully met all indicators, students with high self-efficacy experienced moments of procedural doubt when determining the order of operations for reducing algebraic expressions. This finding is important because it shows that high self-efficacy does not mean being free from cognitive obstacles, but rather is reflected in the ability to persevere and not give up when those obstacles arise, in line with the

strength dimension in the theory Bandura (1997). The dynamics of such processes are generally not documented in quantitative research, which reports only the final results.

Several limitations should be noted when interpreting these findings. As a single-case study, these findings describe a single cognitive profile and are not intended to be generalized to all students categorized as having high self-efficacy. The coding process was also conducted by a single researcher without testing for intercoder agreement; therefore, the interpretation of the units of analysis may have been influenced by the researcher's subjectivity, despite being supported by methodological triangulation. Further research involving multiple subjects or a comparative design across self-efficacy categories is needed to test whether the cognitive mechanisms identified in students with high self-efficacy apply more broadly.

Overall, this study makes three major contributions: it uncovers the cognitive mechanisms that explain the relationship between high self-efficacy and conceptual understanding—a qualitative aspect that quantitative approaches cannot capture; it demonstrates that the attainment of conceptual understanding is a dynamic process punctuated by doubts that are overcome through persistence, rather than a linear process; and providing evidence that understanding of material at the arithmetic-algebra transition is facilitated by students' self-efficacy, with direct implications for classroom teaching practices.

Implications for learning

Based on the cognitive mechanisms described above, several pedagogical implications can be formulated. The finding that students with high self-efficacy achieve conceptual understanding through a process of trying, evaluating, and correcting their own approaches indicates the importance of allowing students to resolve procedural uncertainties independently before the teacher intervenes. Corrections made too quickly have the potential to disrupt the self-regulation process, which is a source of self-efficacy reinforcement (Bandura, 1997). A gradual scaffolding strategy, such as the probing-prompting model, which fosters understanding through guiding questions rather than direct answers (Hanifa et al., 2023), can help students retrace their own thought processes, just as students with high self-efficacy do when faced with uncertainty in algebraic subtraction.

The pattern observed among students with high self-efficacy—who consistently represent real-world situations using algebraic symbols before verifying their results through value substitution—underscores the importance of instruction that emphasizes the translation between concrete, verbal, and symbolic representations (Jupri et al., 2014). It is not merely a matter of repetitive procedural practice. Teachers can design tasks that explicitly require students to translate contextual problems into algebraic form and vice versa, so that students with moderate or low self-efficacy are also trained to build bridges between representations—a common source of difficulty in this subject matter. (Isfayani, 2023; Herawati & Kadarisma, 2021).

Because high self-efficacy has been shown to support perseverance—students with high self-efficacy are better able to cope with procedural uncertainty—teachers need to design gradual mastery experiences for students with moderate and low self-efficacy, for example through graded tasks accompanied by feedback that builds self-confidence. (Pratiwi & Imami, 2022; Sardin et al., 2025). Thus, the development of self-efficacy and the understanding of algebraic concepts can be designed to go hand in hand, rather than as two separate aspects of learning.

Conclusion

This study shows that students with high self-efficacy in the cases examined possess a relational and meaningful understanding of mathematical concepts related to algebraic elements and operations, as reflected in their ability to reconstruct the meaning of concepts in their own words, logically analyze the essential properties of concepts, and systematically solve algebraic operations while explaining the reasoning behind each step. The novelty of this study lies in its in-depth exploration of the cognitive processes underlying these achievements through a case study approach, including the finding that high self-efficacy does not eliminate procedural uncertainty but is instead reflected in students' persistence in overcoming it—a nuanced aspect of the process rarely revealed in previous quantitative studies. As a single-case study with a test instrument limited to three questions, these findings are not intended to be generalized but rather serve as an initial profile that can serve as a starting point for further research involving a more diverse sample of participants across various categories of self-efficacy and educational levels. These findings confirm that strengthening self-efficacy and developing conceptual understanding of mathematics need to be designed as mutually reinforcing processes, rather than two separate aspects of learning.

Conflict of Interest

The authors declare that there are no conflicts of interest.

Auhor Contributions

N.A. conceptualized the research idea presented, collected and organized the data, and prepared the initial draft of the manuscript. M. actively contributed to the development of the methodology, data analysis, and discussion of the findings. S.N.H.H. contributed to the development of the theoretical framework, revision of the manuscript, validation of the results, and approval of the final version of the work. All authors confirm that they have read and approved the final version of this manuscript. The percentage contributions for the conceptualization, drafting, analysis, and revision of this paper are as follows: N.A.: 30%, M.: 20%, S.N.H.H.: 50%.

Data Availability Statement

The authors state that the data supporting the findings of this study will be made available by the corresponding author, S.N.H.H., upon reasonable request.

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