



<https://doi.org/10.51574/kognitif.v6i2.4815>

## The Role of Reflective Thinking in Enhancing Mathematical Problem-Solving Skills

Valeria Zebua, Netti Kariani Mendrofa , Ratna Natalia Mendrofa , Sadiana Lase

**How to cite:** Zebua, V., Mendrofa, N. K., Mendrofa, R. N., & Lase, S. (2026). The Role of Reflective Thinking in Enhancing Mathematical Problem-Solving Skills. *Kognitif: Jurnal Riset HOTS Pendidikan Matematika*, 6(2), 474–485. <https://doi.org/10.51574/kognitif.v6i2.4815>

To link to this article: <https://doi.org/10.51574/kognitif.v6i2.4815>



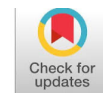
Opened Access Article



Published Online on 26 April 2026



Submit your paper to this journal



## The Role of Reflective Thinking in Enhancing Mathematical Problem-Solving Skills

Valeria Zebua<sup>1\*</sup>, Netti Kariani Mendrofa<sup>1</sup> , Ratna Natalia Mendrofa<sup>1</sup> , Sadiana Lase<sup>1</sup>

<sup>1</sup>Department of Mathematics Education, Faculty of Teacher Training and Education, Universitas Nias

### Article Info

#### Article history:

Received Mar 07, 2026

Accepted Apr 24, 2026

Published Online Apr 26, 2026

#### Keywords:

Reflective Thinking

Mathematical Problem Solving

Problem-solving Skills

Matrix learning

Students' Cognitive Processes

### ABSTRACT

This study was motivated by preliminary findings suggesting that students' mathematical problem-solving skills were deficient. Students encountered challenges in comprehending problem material, formulating solution strategies, executing problem-solving procedures, and evaluating their responses. This study sought to elucidate the function of reflective thinking in enhancing students' mathematical problem-solving capabilities concerning matrices. A descriptive qualitative methodology was utilised, employing 22 eleventh-grade students as participants in the research. Data were gathered by observation, assessments, interviews, and documentation, and were analysed through data reduction, data presentation, and conclusion formulation. The results indicated disparities in students' mathematical problem-solving proficiency across four tiers: extremely high (3 students; mean score = 85.4), high (4 students; mean score = 68.2), moderate (7 students; mean score = 51.7), and low (8 students; mean score = 29.9). Students possessing advanced reflective thinking skills demonstrated a profound understanding of problems, the capacity to identify suitable techniques, rectify errors, and articulate their thought processes eloquently and logically. The findings suggest that reflective thinking significantly enhances students' mathematical problem-solving skills. As students' reflective thinking skill increases, their capacity to comprehend challenges, devise and execute suitable techniques, and independently, critically, and systematically assess and enhance their solutions improves.



This is an open access under the CC-BY-SA licence



### Corresponding Author:

Valeria Zebua,

Department of Mathematics Education,

Faculty of Teacher Training and Education,

Universitas Nias,

Yos Sudarso Street No. 118/E-S, Ombolata Ulu, Gunungsitoli City, North Sumatra 22812, Indonesia.

Email: [zebuavaleria8@gmail.com](mailto:zebuavaleria8@gmail.com)

## Introduction

The ability to solve mathematical problems is a fundamental aspect of mathematics education, since it allows students to comprehend information, choose suitable strategies, implement mathematical procedures, and assess the accuracy of their results (Kholid, Putri, et

al., 2022; Utami et al., 2024; Voica et al., 2020). Polya's problem-solving framework delineates this process into four primary stages: comprehending the problem, formulating a strategy, executing the strategy, and reflecting on the outcome (Abbeduto et al., 2019; Sindy Mustika Sari et al., 2022). The stages indicate that problem solving is not solely a procedural task, but also a contemplative process necessitating students to oversee and assess their cognitive processes during mathematical endeavours.

Initial observations at the research site suggested that students' mathematical problem-solving skills were inadequately developed. A significant number of students struggled to autonomously recognise issues and formulate suitable answers for the mathematics assignments provided by the instructor. This condition indicates that students continue to encounter significant challenges in applying the mathematical knowledge and skills they have acquired. Despite obtaining explanations or advice from the instructor, students continued to struggle with independently solving mathematical problems and frequently failed to exhibit a profound comprehension of the mathematical topics being taught. This scenario underscores the necessity for a more efficacious pedagogical strategy to enhance students' capacity for independent mathematical problem-solving (Goulet-Lyle et al., 2020; Kim et al., 2022).

Interviews with the mathematics instructor indicated that during the learning process, students exhibited less engagement and failed to completely concentrate on the instructor's explanations. Consequently, they encountered challenges in resolving issues that necessitated investigation and profound comprehension. Certain students depended predominantly on rote memorisation of formulas, lacking comprehension of the underlying processes and rationale for their application. This circumstance resulted in a mechanical approach to mathematics education, when students concentrated solely on final results, neglecting the necessary solution procedures. As a result, students had challenges while addressing assignments that necessitated mathematical problem-solving skills. This condition was also apparent when the prospective researcher conducted a mathematics problem-solving assessment for Grade XI students. The findings indicated that students lacked a comprehensive understanding of the indications of mathematical problem-solving proficiency.

The prospective researcher determined that the kids' mathematics problem-solving abilities were still deficient based on their responses. This was evident in multiple phases of problem-solving. Initially, many students struggled to discern the material inside the questions, hindering their ability to establish suitable response strategies (Vicente et al., 2022). Secondly, while formulating a solution strategy, students frequently neglected to select or identify an effective method, leading to ineffective problem-solving (Muzaini et al., 2023). Third, while implementing the solution, some students initiated the process but frequently faced challenges in applying the appropriate concepts or methods (Szabo et al., 2020). Fourth, during the retrospective phase, numerous students failed to reassess their responses, resulting in unreported errors and a lack of corrections (Wilkie, 2025). The inadequate mathematical problem-solving skills of students indicate the necessity for enhancing the learning process via a more effective methodology.

If this issue remains unaddressed, it may have enduring consequences for the quality of mathematical education. Inadequate mathematical problem-solving skills might hinder students' overall academic performance and their preparedness to confront workplace obstacles. Students who struggle to answer mathematical issues proficiently may also find it challenging to apply the technical abilities acquired in their vocational disciplines. Consequently, it is essential to determine a strategy that aids students in surmounting challenges in mathematical problem-solving.

A potential option is to cultivate students' reflective thinking. Başol & Gencil, (2013) asserts that reflective thinking is a superior cognitive skill that significantly contributes to the

enhancement of other advanced thinking abilities. Advanced mathematical cognition, encompassing analysis, problem-solving, reasoning, and intricate mathematical representation, is significantly reliant on reflective thinking. Consequently, reflective thinking ought to be cultivated and implemented in mathematical education (Kholid, Swastika, et al., 2022). Thahir et al. (2019) elucidates that reflective thinking is a competency that empowers students to integrate diverse knowledge forms, facilitating the formulation and resolution of novel challenges using pertinent methodologies tailored to the specific context (Ghanizadeh, 2017; Kholid, Putri, et al., 2022; Sormunen et al., 2020). This capability encompasses not just comprehending given information but also analysing and formulating appropriate solutions through the integration of pertinent views. Consequently, reflective thinking can enhance and expand students' comprehension of mathematics.

Reflective thinking in vocational mathematics education can be enhanced through diverse methods, including group discussions, reflective journals, and chances for students to articulate their problem-solving processes (Evin Gencel & Saracaloğlu, 2018). In this process, the educator serves as a facilitator, assisting students in analysing and reflecting on their actions, while also offering constructive comments. Students' mathematical reflective thinking encompasses the capacity to integrate mathematical concepts based on prior knowledge and experience. This capability is crucial for facilitating the advancement of mathematical learning. Consequently, when students cultivate reflective thinking effectively, they can enhance their comprehension of mathematical topics and utilise them in diverse contexts. This circumstance can enhance the entire mathematics learning experience. Reflective thinking enables students to evaluate the best suitable strategy prior to addressing a problem. A reflective approach enables students to examine and resolve issues more profoundly, rather than simply seeking the perfect solution. Kholid, Putri, et al. (2022) assert that reflective thinking is crucial for both educators and learners in mathematics education focused on problem-solving skills. Furthermore, Evin Gencel & Saracaloğlu (2018) elucidate that students' mathematical reflective thinking encompasses their capacity to integrate mathematical concepts derived from their prior knowledge and experiences.

This method promotes more engaged and significant learning. Students concentrate not only on the ultimate outcome but also on the methodology employed to attain that outcome. Consequently, cultivating reflective thinking is beneficial for enhancing students' reflective abilities and may also elevate the quality of mathematics education in vocational institutions. It offers a more thorough and practical method for assisting students in comprehending, resolving, and assessing mathematical issues.

## Method

### Research Design

This study employed a descriptive qualitative approach to examine students' reflective thinking in mathematical problem solving, particularly in the topic of matrices. This design was appropriate because the study focused on describing students' problem-solving processes, identifying the characteristics of their reflective thinking, and interpreting how students understood, planned, implemented, and reviewed their solutions. The study did not aim to test relationships between variables or compare groups statistically. Instead, it sought to provide an in-depth description of the role of reflective thinking in supporting students' mathematical problem-solving ability.

## Participants

The participants were 22 eleventh-grade students from a vocational high school. They were selected through purposive sampling based on specific criteria. The selected students had participated in mathematics learning on matrices and had been introduced to reflective thinking activities. The participants represented different levels of mathematical problem-solving ability, allowing the researcher to explore how reflective thinking appeared across varied student responses.

## Instruments

Data were collected using observation sheets, a written mathematical problem-solving test, interview guidelines, and documentation. The observation sheet was used to record students' learning activities, participation, responses, and problem-solving behavior during mathematics instruction. The written test consisted of three essay questions on matrices and was designed based on the stages of mathematical problem solving: understanding the problem, devising a plan, carrying out the plan, and looking back. The test instrument was examined for validity and reliability before being used in the study. The item validity test showed that all three items were valid, with correlation coefficients exceeding the critical value of 0.444 at the 5% significance level for  $N = 20$ . The reliability test using the alpha formula produced a coefficient of 0.9066, indicating a high level of internal consistency. Thus, the written test was considered suitable for collecting data on students' mathematical problem-solving ability. The interview guideline was used to obtain deeper information about students' reflective thinking during the problem-solving process. The interviews helped clarify students' written answers and reveal the reasoning behind their strategies, errors, and revisions. Documentation, including students' written work and observation notes, was used to support and verify the data obtained from observation, tests, and interviews.

## Data Collection Procedure

The data collection process began with classroom observation during mathematics learning. The observation focused on students' engagement, responses, and difficulties in solving matrix-related problems. After the learning process had been completed, students were given a written test consisting of essay questions. The test was used to identify students' mathematical problem-solving ability based on the four problem-solving stages. After the test, students' answers were assessed using a scoring rubric aligned with the problem-solving indicators. The results were then used to classify students' levels of mathematical problem-solving ability. Selected students were interviewed to explore their reflective thinking processes in greater depth. The interview data were used to explain students' written responses, especially how they understood the problem, selected strategies, applied procedures, identified errors, and reviewed their answers. Data from observation, written tests, interviews, and documentation were compared through triangulation. This process was used to check the consistency of findings across different data sources and to strengthen the credibility of the research results.

## Data Analysis

The data were analyzed using three main stages: data reduction, data display, and conclusion drawing. In the data reduction stage, the researcher selected, organized, categorized, and focused the data that were relevant to the research objective. Students' written answers and interview responses were categorized based on the stages of mathematical problem solving and the indicators of reflective thinking. In the data display stage, the reduced data were presented

in descriptive and narrative forms. The presentation focused on students' problem-solving processes, reflective thinking characteristics, and the connection between students' responses and the problem-solving indicators. This stage allowed the researcher to identify patterns in how students understood problems, planned strategies, carried out procedures, and reviewed their solutions. In the conclusion drawing stage, the researcher reviewed the displayed data, checked the consistency of emerging patterns, and verified the findings using evidence from observation, written tests, interviews, and documentation. The final conclusions were drawn based on factual data obtained from the field. Through this process, the study described the role of reflective thinking in supporting students' mathematical problem-solving ability in the topic of matrices.

## Research Findings

### Results of the Mathematical Problem-Solving Ability Test

The mathematical problem-solving ability test was administered to 22 Grade XI students. The test aimed to describe students' levels of ability in solving mathematical problems related to matrices. Based on the test results, students' abilities were distributed across four categories: low, moderate, high, and very high.

**Table 1.** Descriptive Statistics of Students' Mathematical Problem-Solving Ability Test Results

| Aspect             | Score    |
|--------------------|----------|
| Number of students | 22       |
| Lowest score       | 22.9     |
| Highest score      | 91.7     |
| Mean               | 57.02    |
| Median             | 56.25    |
| General category   | Moderate |

Table 1 shows that the mean score of students' mathematical problem-solving ability was 57.02, placing it in the moderate category. The highest score was 91.7, while the lowest score was 22.9. These results indicate that students' mathematical problem-solving ability varied considerably and had not yet developed evenly.

**Table 2.** Distribution of Students Based on Mathematical Problem-Solving Ability Categories

| Category  | Mean score per category | Number of students | Percentage |
|-----------|-------------------------|--------------------|------------|
| Very low  | 0                       | 0                  | 0%         |
| Low       | 30.37                   | 7                  | 31.8%      |
| Moderate  | 54.75                   | 7                  | 31.8%      |
| High      | 78.15                   | 4                  | 18.2%      |
| Very high | 86.47                   | 4                  | 18.2%      |

Table 2 shows that the most dominant categories were low and moderate, each consisting of 7 students, or 31.8%. Meanwhile, 4 students, or 18.2%, were in the high category, and another 4 students, or 18.2%, were in the very high category. No students were classified in the very low category. These findings indicate that most students had not yet reached an optimal level of mathematical problem-solving ability. In general, the test results show that students already had a basic ability to understand and solve mathematical problems, but they still showed weaknesses in accuracy, completeness of solution steps, and reviewing their answers.

## Results of the Teacher Interview

The teacher interview was conducted to obtain information about the teacher's views on reflective thinking and students' mathematical problem-solving ability. The interview results showed that the teacher considered both abilities important in mathematics learning, especially in vocational education.

**Table 3. Excerpts from the Teacher Interview**

| Focus                                | Interview excerpt   | Meaning of the finding  |
|--------------------------------------|---|---|
| Understanding of reflective thinking | "Reflective thinking is students' ability to review the steps used in solving problems, evaluate errors, and improve their strategies."                   | The teacher understood reflective thinking as a process of evaluating and improving one's thinking. |
| Importance of problem solving        | "Mathematical problem solving is the skill of finding solutions to contextual problems that are often encountered in the workplace or in real life."      | The teacher viewed problem solving as an applicable ability, not merely computational skill.        |
| Learning strategies                  | "I provide contextual problems related to students' vocational fields and guide them through discussion and hands-on practice."                           | The teacher attempted to connect mathematics with students' real-life contexts.                     |
| Learning challenges                  | "Some students think mathematics is difficult and lack confidence. In addition, differences in students' abilities and limited time are also challenges." | The main barriers were students' low confidence, heterogeneous ability levels, and limited time.    |
| Efforts to overcome difficulties     | "I divide students into learning groups, provide gradual instruction, use visual media, and give positive encouragement."                                 | The teacher applied differentiation and gradual support to help students.                           |

Based on the interview, the teacher understood that reflective thinking and mathematical problem-solving ability were closely related. The teacher also believed that learning would be more effective when mathematics was connected to real-life and workplace contexts. However, the teacher still faced several challenges, including differences in students' abilities, limited instructional time, and students' low confidence in mathematics.

## Results of the Student Interviews

Student interviews were conducted to explore how students understood problems, selected solution strategies, evaluated their work, and explained the reasons behind their answers. The interview results showed clear differences among students across different ability categories.

**Table 4. Excerpts from Student Interviews Based on Ability Categories**

| Subject   | Category  | Interview excerpt  | Meaning of the finding   |
|-----------|-----------|--|--|
| Subject 1 | Very high | "I read the problem, wrote down what was known and what was asked, and then chose the appropriate formula or strategy."<br>"I always make sure that my answer is logical and consistent with the concept." | The student understood the initial stage of problem solving systematically.<br>The student evaluated and reviewed the answer reflectively.     |
| Subject 2 | High      | "I checked the formula and the calculation steps again."<br>"Sometimes I can find another way besides the method taught by the teacher."   | The student showed a habit of reviewing the solution procedure.<br>The student began to show strategic flexibility, although not consistently. |

| Subject   | Category | Interview excerpt   | Meaning of the finding  |
|-----------|----------|---|---|
| Subject 3 | Moderate | “I reread the problem several times until I understood it.”<br>“I can only use the method taught by the teacher.”   | The student needed repetition to understand the problem.<br>The student tended to depend on one strategy and had not yet shown flexibility. |
| Subject 4 | Low      | “I often get confused, and I usually wait for the teacher’s explanation.”<br>“I only read it once and then guessed the answer.”                                     | The student had not yet developed independence in understanding problems.<br>The student did not yet have an adequate analytical strategy.  |
| Subject 5 | Low      | “I usually do not evaluate my answer. I only write based on an example if there is one.”<br>“I cannot explain it because I do not really understand my own method.” | The student had not yet shown reflection on the answer.<br>The student was not yet able to explain the reasoning behind the solution.       |

Based on Table 4, students in the very high and high categories tended to understand the problems, determine appropriate strategies, review their solution steps, and explain the reasons behind their answers. In contrast, students in the moderate category were still limited to one strategy and had not optimally evaluated their own work. Meanwhile, students in the low category tended to be passive, guessed answers, did not review their work, and were unable to explain their thinking processes.

### Triangulation of Test and Interview Results

Method triangulation was conducted by comparing the mathematical problem-solving test results with the student interview results. This comparison aimed to examine the consistency between students’ scores and the characteristics of their thinking processes when solving problems.

**Table 5. Triangulation of Test and Interview Results**

| Category  | Test result findings  | Supporting interview excerpt  | Interpretation   |
|-----------|---|---|--|
| Very high | Students were able to write what was known and what was asked, organize the solution steps coherently, and solve the problems well. | “I checked the steps one by one and compared them with the concepts taught by the teacher.” | High scores were supported by strong reflective ability, clear strategies, and answer evaluation.                    |
| High      | Students understood the basic concepts and could solve the problems, although some weaknesses remained in completeness or accuracy. | “I checked the formula and the calculation steps again.”                                    | Students already showed reflective processes, but they were not yet consistent in accuracy and strategy development. |
| Moderate  | Students were able to begin the solution process, but their answers were incomplete and tended to rely on only one method.          | “I can only use the method taught by the teacher.”  | Students’ problem-solving ability was adequate, but reflection and strategic flexibility remained limited.           |
| Low       | Students did not complete the solution steps fully, worked less systematically, and did not check their answers.                    | “I only read it once and then guessed the answer.”  | Low scores were related to weak problem understanding, limited strategy use, and poor self-evaluation.               |

The triangulation results showed that students' test achievements were consistent with the interview findings. Students in the very high and high categories showed more systematic, reflective, and confident thinking processes. Students in the moderate category had begun to understand the basic problem-solving steps, but they were not yet consistent in evaluating their answers and developing alternative strategies. Meanwhile, students in the low category had not yet demonstrated adequate reflective thinking processes, which made it difficult for them to understand problems and complete the solutions.

### Synthesis of the Research Findings

Based on the test results, teacher interview, student interviews, and method triangulation, students' mathematical problem-solving ability was generally in the moderate category, with a mean score of 57.02. Most students were still in the low and moderate categories, indicating that their mathematical problem-solving ability had not yet developed optimally. The interview results showed that students in the very high and high categories demonstrated stronger characteristics of reflective thinking. They were able to read problems carefully, identify important information, choose appropriate strategies, review their answers, and explain the reasons behind the steps they took. In contrast, students in the moderate and low categories did not yet show adequate reflection. They tended to depend on examples, use only one method, show limited evaluation of their answers, and, in some cases, guess the results without clear analysis. These findings indicate that students with stronger reflective thinking tended to demonstrate better mathematical problem-solving ability. Therefore, mathematics learning should provide more opportunities for students to examine their solution steps, evaluate errors, and explain the reasoning behind their answers.

**Table 6.** Characteristics of Mathematical Problem-Solving Ability Based on Category

| Category  | Understanding the problem | Devising a solution plan | Carrying out the solution | Looking back |
|-----------|---------------------------|--------------------------|---------------------------|--------------|
| Very high | Very good                 | Very good                | Good                      | Good         |
| High      | Good                      | Good                     | Fairly good               | Fairly good  |
| Moderate  | Fair                      | Fair                     | Incomplete                | Poor         |
| Low       | Poor                      | Poor                     | Poor                      | Very poor    |

### Discussion

These findings demonstrate a strong correlation between students' quantitative problem-solving skills and their reflective thinking. Students in the uppermost and high categories demonstrated the ability to comprehend the problem, choose suitable techniques, assess their solution processes, recognise alternate solutions, and articulate their reasoning with assurance. Conversely, students in the middle category predominantly depended on a singular approach imparted by the instructor, whilst students in the low category exhibited passivity, resorted to conjecture without thorough investigation, and failed to independently assess their cognitive processes. This pattern indicates that reflective thinking operates as a cognitive control mechanism that assists students in monitoring, justifying, and revising their mathematical tasks (DiNapoli & Miller, 2022; Son & Lee, 2021; Voica et al., 2020).

The findings align with Kholid, Putri, et al. (2022) Kholid, Swastika, et al. (2022) and Thahir et al. (2019), who asserted that reflective thinking is a metacognitive process allowing students to evaluate their cognitive processes and problem-solving methods. Through this approach, students may recognise faults and devise more efficient ways for resolving mathematical problems (Ghanizadeh, 2017). This study expands this perspective by

demonstrating that reflective thinking is pertinent not just at the last phase of answer verification but also facilitates every stage of problem-solving, encompassing problem comprehension, plan formulation, solution execution, and retrospective evaluation (Sormunen et al., 2020). Consequently, reflective thinking seems to enhance students' capacity to progress from rote application of formulas to more analytical and methodical problem-solving.

The results corroborate Evin Gencil & Saracaloğlu (2018) assertion that reflective thinking facilitates deep information processing, comprehension of interrelations among mathematical concepts, and self-evaluation of one's work. This capability is crucial in matrix problems as students must interpret symbolic information, select appropriate processes, and assess the consistency of their solutions (Kholid, Putri, et al., 2022). Students exhibiting enhanced reflective thinking were more adept at linking prior knowledge to the problem context, while those with diminished reflective thinking relied on examples and methods without comprehending the fundamental principles.

This study enhances mathematics education by demonstrating that reflective thinking can be a crucial framework for comprehending variations in students' mathematical problem-solving capabilities (Thahir et al., 2019). The findings indicate that challenges in problem-solving arise not solely from inadequate procedural knowledge, but also from poor self-monitoring, restricted strategy appraisal, and insufficient error reflection. Consequently, mathematics education should offer increased opportunity for students to articulate their reasoning, contrast various methodologies, recognise errors, and substantiate their solution processes.

From an educational standpoint, educators can enhance students' reflective thinking via contextual problem-solving assignments, group discussions, reflective journals, and problem-based learning. In matrix learning, students may be instructed to document their knowledge and enquiries, elucidate the rationale behind certain procedures, assess the reasonableness of their responses, and amend erroneous solution stages (Figueroa et al., 2018). These activities can enhance students' independence, critical thinking, and systematic approaches to solving mathematical issues, especially in vocational situations where mathematics is anticipated to facilitate real-life and workplace problem-solving. Due to the study's focus on a restricted cohort of students inside a singular vocational school setting, the results should be regarded as a detailed account rather than a foundation for extensive generalisation. Future research may use a larger sample size, diverse mathematical subjects, or intervention-based methodologies to investigate the systematic development of reflective thinking aimed at enhancing students' mathematical problem-solving skills.

## Conclusion

Reflective thinking significantly enhances students' mathematical problem-solving capabilities. Reflective thinking enables students to comprehend challenges more profoundly, discern pertinent information, and integrate it with their existing knowledge to formulate suitable solution solutions. Students exhibiting enhanced reflective thinking were more adept at solving mathematical problems effectively, as they could identify appropriate strategies, systematically apply solution stages, and assess the validity of their answers. The results indicate that reflective thinking enhances students' critical evaluation of their problem-solving methods. Students exhibiting superior reflective thinking had enhanced abilities in error identification, procedural revision, and conclusion formulation. Conversely, students exhibiting diminished reflective thinking tended to depend on examples, employ restricted tactics, and demonstrate reduced autonomy in assessing their responses.

Consequently, mathematics education should offer increased opportunity for students to articulate their reasoning, evaluate their solution processes, and contemplate errors, thereby rendering problem-solving activities more significant and methodical.

### Conflict of Interest

The authors declare no conflict of interest.

### Author Contributions

V.Z. conceived the research idea presented and collected the data. N.K.M. actively participated in the development of the theory, organization and division of tasks. R.N.M. and S.L. actively participated in the development of the methodology, organization and analysis of the data, discussion of the results and approval of the final version of the work. All authors declare that the final version of this paper has been read and approved. The total percentage of contributions to the conceptualization, preparation, and correction of this paper is as follows: V.Z.: 60%, N.K.M.: 20%, R.N.M.: 10% and S.L.: 30%

### Data Availability Statement

The authors state that the data supporting the findings of this study will be made available by the corresponding author, [V.Z.], upon reasonable request.

### References

- Abbeduto, L., Murphy, M. M., Richmond, E. K., Amman, A., Beth, P., Weissman, M. D., Kim, J.-S., Cawthon, S. W., Karadottir, S., Abe, H., Yada, H., Yamamoto, T., Sohma, H., Abraham, S., Perez, P., Adarves-Yorno, I., Jetten, J., Postmes, T., Haslam, S. A., ... Elaad, E. (2019). Teacher perceptions of two multi-component interventions: Disability awareness and science. *Frontiers in Psychology, 31*(2).
- Başol, G., & Gencil, I. E. (2013). Reflective thinking scale: A validity and reliability study. *Kuram ve Uygulamada Eğitim Bilimleri, 13*(2), 941–946.
- DiNapoli, J., & Miller, E. K. (2022). Recognizing, supporting, and improving student perseverance in mathematical problem-solving: The role of conceptual thinking scaffolds. *Journal of Mathematical Behavior, 66*. <https://doi.org/10.1016/j.jmathb.2022.100965>
- Evin Gencil, I., & Saracaloğlu, A. S. (2018). The Effect of Layered Curriculum on Reflective Thinking and on Self-Directed Learning Readiness of Prospective Teachers. *International Journal of Progressive Education, 14*(1), 8–20. <https://doi.org/10.29329/ijpe.2018.129.2>
- Figuroa, A. P., Possani, E., & Trigueros, M. (2018). Matrix multiplication and transformations: an APOS approach. *Journal of Mathematical Behavior, 52*(October 2017), 77–91. <https://doi.org/10.1016/j.jmathb.2017.11.002>
- Ghanizadeh, A. (2017). The interplay between reflective thinking, critical thinking, self-monitoring, and academic achievement in higher education. *Higher Education, 74*(1), 101–114. <https://doi.org/10.1007/s10734-016-0031-y>
- Goulet-Lyle, M. P., Voyer, D., & Verschaffel, L. (2020). How does imposing a step-by-step solution method impact students' approach to mathematical word problem solving? *ZDM - Mathematics Education, 52*(1), 139–149. <https://doi.org/10.1007/s11858-019-01098-w>
- Kholid, M. N., Putri, Y. P., Swastika, A., Maharani, S., & Ikram, M. (2022). What are the pupils' challenges in implementing reflective thinking for problem-solving? *AIP Conference Proceedings, 2479*(July). <https://doi.org/10.1063/5.0099600>

- Kholid, M. N., Swastika, A., Ishartono, N., Nurcahyo, A., Lam, T. T., Maharani, S., Ikram, M., Murniasih, T. R., Majid, Wijaya, A. P., & Pratiwi, E. (2022). Hierarchy of Students' Reflective Thinking Levels in Mathematical Problem Solving. *Acta Scientiae*, 24(6), 24–59. <https://doi.org/10.17648/acta.scientiae.6883>
- Kim, N. J., Vicentini, C. R., & Belland, B. R. (2022). Influence of Scaffolding on Information Literacy and Argumentation Skills in Virtual Field Trips and Problem-Based Learning for Scientific Problem Solving. *International Journal of Science and Mathematics Education*, 20(2), 215–236. <https://doi.org/10.1007/s10763-020-10145-y>
- Muzaini, M., Rahayuningsih, S., Ikram, M., & Nasiruddin, F. A. Z. (2023). Mathematical Creativity: Student Geometrical Figure Apprehension in Geometry Problem-Solving Using New Auxiliary Elements. *International Journal of Educational Methodology*, 9(1), 139–150. <https://doi.org/10.12973/ijem.9.1.139>
- Sindy Mustika Sari, Aldi Firmansyah, & Reza Lestari. (2022). Analisis Kesulitan Siswa Dalam Menyelesaikan Soal Matematika Problem Solving Berdasarkan Tahapan Heuristik Polya Siswa Kelas XI Ipa. *Seminar Nasional Pendidikan Guru Sekolah Dasar Universitas Kuningan*, 2(1), 109–114. <https://doi.org/10.25134/prosidingsemnaspgsd.v2i1.32>
- Son, J. W., & Lee, M. Y. (2021). Exploring the Relationship Between Preservice Teachers' Conceptions of Problem Solving and Their Problem-Solving Performances. *International Journal of Science and Mathematics Education*, 19(1), 129–150. <https://doi.org/10.1007/s10763-019-10045-w>
- Sormunen, K., Juuti, K., & Lavonen, J. (2020). Maker-Centered Project-Based Learning in Inclusive Classes: Supporting Students' Active Participation with Teacher-Directed Reflective Discussions. *International Journal of Science and Mathematics Education*, 18(4), 691–712. <https://doi.org/10.1007/s10763-019-09998-9>
- Szabo, Z. K., Körtesi, P., Guncaga, J., Szabo, D., & Neag, R. (2020). Examples of problem-solving strategies in mathematics education supporting the sustainability of 21st-century skills. *Sustainability (Switzerland)*, 12(23), 1–28. <https://doi.org/10.3390/su122310113>
- Thahir, A., Komarudin, Hasanah, U. N., & Rahmahwaty. (2019). MURDER learning models and self efficacy: Impact on mathematical reflective thinking ability. *Journal for the Education of Gifted Young Scientists*, 7(4), 1120–1133. <https://doi.org/10.17478/jegys.594709>
- Utami, A. M., Adelia, R., Kurniawati, I., Damayanti, E., & Damara, T. D. (2024). Implementasi Permainan Tic-Tac-Toe sebagai Konteks Pembelajaran Matematika pada Materi Keliling Bangun Datar dalam Menanamkan Kemampuan Pemecahan Masalah Siswa Sekolah Dasar. 08(December 2023), 180–188.
- Vicente, S., Verschaffel, L., Sánchez, R., & Múñez, D. (2022). Arithmetic word problem solving. Analysis of Singaporean and Spanish textbooks. *Educational Studies in Mathematics*, 111(3), 375–397. <https://doi.org/10.1007/s10649-022-10169-x>
- Voica, C., Singer, F. M., & Stan, E. (2020). How are motivation and self-efficacy interacting in problem-solving and problem-posing? *Educational Studies in Mathematics*, 105(3), 487–517. <https://doi.org/10.1007/s10649-020-10005-0>
- Wilkie, K. J. (2025). 'Things haven't gone the way I thought they'd go': secondary mathematics teachers talk about trialling a problem-solving pedagogy. *Journal of Mathematics Teacher Education*. <https://doi.org/10.1007/s10857-025-09709-y>

## Author Biography

|   |  |
|---|--|
|    | <p><b>Valeria Zebua</b> was born in Hilimbaruzo, North Sumatra, on June 29, 2002. She is the second of eight children of Foarota Zebua and Rosmei Bu'ulolo. She completed her elementary education in 2014 at SD Negeri 078011 Tetelesi. She then continued her studies at SMP Negeri 2 Alasa and graduated in 2017. After completing junior high school, she continued her education at SMK Negeri 1 Alasa. In 2021, she pursued higher education at a private higher education institution in Nias, namely the Institute of Teacher Training and Education (IKIP) Gunungsitoli, which became Universitas Nias in 2022. She enrolled in the Mathematics Education Study Program, Faculty of Teacher Training and Education. Email: <a href="mailto:zebuavaleria8@gmail.com">zebuavaleria8@gmail.com</a></p> |
|    | <p><b>Netti Kariani Mendrofa</b> is a lecturer and researcher at the Department of Mathematics Education, Faculty of Teacher Training and Education, Universitas Nias, North Sumatra, Indonesia. Her research interests include Mathematical Problem Solving, and Technology-Assisted Learning. Affiliation: University Nias, Email: <a href="mailto:netti.mend14@gmail.com">netti.mend14@gmail.com</a></p>  |
|   | <p><b>Ratna Natalia Mendrofa</b> is a lecturer in Department of Mathematics Education, Faculty of Teacher Training and Education, Universitas Nias. She holds a master's degree from Universitas Negeri Padang. Her current research interests include mathematical disposition, problem-solving skills, and the use of technology in learning. Email: <a href="mailto:ratnamend@gmail.com">ratnamend@gmail.com</a></p>  |
|  | <p><b>Sadiana Lase</b> is a lecturer in the Department of Mathematics Education, Faculty of Teacher Training and Education, Universitas Nias. She holds a master's degree from Universitas Negeri Padang. Her research interests focus on mathematical disposition, problem-solving skills, and the integration of technology in learning. Email: <a href="mailto:sadianalase01@gmail.com">sadianalase01@gmail.com</a></p>   |