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# The Role of Cognitive Flexibility in Resolving Student Misconceptions: Evidence from Algebra Tasks

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**ABSTRACT**

Algebra is one of the fundamental subjects in secondary school mathematics education, but various studies show that students still often experience misconceptions, particularly in root form operations, which tend to persist even after formal instruction. One ability that is considered to have the potential to help students respond to these misconceptions is cognitive flexibility, which is the ability to use, compare, and switch between strategies adaptively. This case study aims to identify the characteristics of students' misconceptions in algebraic root operations and describe the role of cognitive flexibility in terms of variety, shifting, and justification/reflection. The study uses a qualitative approach with a contrastive case study design. Data were collected through a written test containing 12 root questions and semi-structured interviews based on stimulated recall. The analysis was conducted by combining mathematical correctness and cognitive flexibility scores with within-case and cross-case qualitative analysis. The results show that cognitive flexibility plays a role in responding to misconceptions, but its role is conditional. The novelty of this study lies in the finding that cognitive flexibility only contributes to the revision of misconceptions when supported by a strong conceptual understanding. Without this foundation, flexibility tends to produce procedural variations without meaningful conceptual change. The implication is that mathematics teachers need to design root form learning that not only encourages the use of various strategies but also emphasizes strategy comparison and conceptual reflection to help students reconstruct their understanding.



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## Introduction

Algebra is one of the main pillars of the secondary school mathematics curriculum and is seen as a "gateway" to advanced mathematics, science, and engineering. However, various studies show that many students still experience fundamental difficulties when dealing with algebraic objects such as variables, root forms, and equations, as reflected in the high number of errors and misconceptions on these topics. Analysis of algebraic errors shows that students often perform procedures that do not follow the rules, for example, incorrectly grouping like terms, simplifying algebraic fractions incorrectly, or applying operations mechanically without adequate conceptual understanding (Moru & Mathunya, 2022). This condition indicates that students' mastery of algebra is not just a matter of "memorizing formulas," but is closely related to how they represent, interpret, and modify symbolic expressions meaningfully.

One area of algebra that consistently causes misconceptions is radical expressions. Research on high school students shows that many students have a mistaken understanding of the definition and operations of radicals, for example, assuming that $\sqrt{(a+b)} = \sqrt{a} + \sqrt{b}$, misinterpreting the root sign for negative numbers, or combining radical forms with different radicals without first simplifying them (Özkan, 2011). Cross-country studies also report that students often simplify $\sqrt{(a^2+b^2)}$ to $a+b$, as if the rule for multiplying radicals ($\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$) also applies to addition within radicals (Al-rababaha et al., 2022; Moru & Mathunya, 2022). Research at various levels shows that students tend to memorize radical definitions and rules superficially without adequate conceptual understanding, so that in new situations they generalize rules incorrectly and produce systematically wrong answer patterns, not just momentary calculation errors (Ozkan & Mehmet, 2012). Furthermore, studies on algebraic misconceptions based on learning styles found that misconceptions do not only appear in one type of learning style; students with certain learning styles can show consistent conceptual errors, such as misunderstanding variables, algebraic operation rules, and symbolic representations, which then affect root form operations (Elma et al., 2025). More recent findings also confirm that misconceptions in exponential and radical expressions are related to students' tendency to work at a "reduced abstraction" level, i.e., sticking to the surface features of expressions (simply looking at the numbers inside the root) without activating deeper conceptual structures such as radicand factorization and equality conditions (Şenay, 2024). This makes misconceptions about root forms persistent and often difficult to change with conventional explanations only.

In the perspective of conceptual change theory, students' misconceptions are viewed not merely as "wrong answers," but as part of a relatively stable and coherent alternative conceptual system for the students themselves. Posner and colleagues emphasize that meaningful learning requires conceptual reconstruction, in which an old, inadequate conception is replaced by a new conception that is more intelligible, plausible, and fruitful for students (Posner et al., 1982). This theory of conceptual change was then developed and applied in the context of mathematics education to explain how students' intuitive ideas about numbers, operations, and symbolic representations can persist even after formal instruction, and what conditions are necessary for a shift in perspective toward a more scientific understanding of mathematics (Liljedahl, 2011). confirm that effective interventions to overcome misconceptions usually do not only present the correct procedures, but also create cognitive conflict, provide opportunities to compare various representations or strategies, and encourage students to reflect on why an old way of thinking is no longer adequate.

In recent years, mathematicians have shifted their attention to the role of cognitive flexibility, or the ability to view problems from various angles, access more than one strategy, and switch adaptively between these strategies. In the context of mathematical flexibility, this ability is defined as knowledge of various ways to solve mathematical problems and the ability to choose the strategy that best suits the characteristics of the problem and the goal of problem solving (Rahaju et al., 2024; Star & Rittle-Johnson, 2008). Studies on procedural flexibility

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show that exposure to and comparison of various algebraic solution strategies, such as several ways to solve equations, can improve students' procedural knowledge and flexibility in choosing strategies (Star et al., 2020). Other research that specifically explores cognitive flexibility in mathematical problem solving shows that more flexible students tend to be able to generate more ideas, change approaches when encountering obstacles, and reflect on the efficiency of the strategies used (Rahayuningsih et al., 2020). However, most of these studies still emphasize the efficiency of problem solving (accuracy and speed) rather than the ability to flexibly correct deep-rooted misconceptions.

When viewed together, the literature on algebraic misconceptions and the literature on cognitive flexibility still tend to run parallel. Studies on radical misconceptions often describe the types of misconceptions, their causes, and several teaching approaches to reduce these errors, but rarely include students' cognitive flexibility profiles as an important variable in the analysis (Özkan, 2011). On the other hand, research on mathematical flexibility and flexible strategy use more often focuses on the relationship between flexibility, accuracy, and problem-solving performance, rather than on how flexibility plays a role in the process of misconception resolution (Star & Rittle-Johnson, 2008). As a consequence, there is still limited empirical evidence that explains in detail how students with different levels of flexibility (e.g., high vs. low) respond to algebra tasks explicitly designed to elicit and challenge misconceptions, especially in the context of root forms. This gap is even more relevant in the Indonesian context, where several studies have captured radical misconceptions, but few have linked them to cognitive flexibility as one of the key factors that may explain why some students are able to revise their concepts while others persist in their erroneous thinking (Pratiwi et al., 2019).

Based on this gap, this study specifically aims to investigate the role of cognitive flexibility in the context of students' misconceptions in algebra tasks, with a focus on root form operations. The study uses diagnostic misconception tasks, strategy flexibility tasks, and adaptability tasks with modified conditions to produce a rich picture of how subjects with different flexibility profiles respond to various types of radical questions. More specifically, the objectives of this study are as follows: (1) To identify students' misconception profiles and characteristics in algebraic radical material through a series of diagnostic tasks; (2) To describe students' cognitive flexibility profiles in terms of variety, shifting, and justification/reflection in completing strategy tasks and adaptability tasks in an algebraic context..

Based on these objectives, this study was formulated into the following research questions: (1) What are the characteristics of students' misconceptions regarding root operations in the given algebra tasks? (2) What is the profile of students' cognitive flexibility in terms of diversity of strategies, shifting abilities, and quality of justification in completing these algebra tasks?.

## Method

### Type of Research and Subjects

This study uses a qualitative approach with a case study design, which aims to gain an in-depth understanding of the role of cognitive flexibility in responding to students' misconceptions about root operations. The case study design was chosen because it allows researchers to examine in detail the thinking processes, strategies, and justifications of students in an authentic context, so that the main focus of this study is depth of analysis, rather than statistical generalization to a wider population (Creswell, 2012).

Subject selection was conducted through purposive sampling with contrasting case sampling, which is the selection of cases that are deliberately contrasted to reveal differences in reasoning patterns on the same phenomenon. Two high school students, coded NH and YH, were selected as contrasting cases from one class based on the following criteria: (1) both showed misconceptions in root operations, (2) had relatively comparable levels of mathematical correctness, but (3) exhibited different cognitive flexibility profiles based on initial flexibility rubric scoring. This approach allows for a more in-depth exploration of the relationship between cognitive flexibility and students' tendencies to maintain or revise misconceptions.

The use of two subjects is considered adequate and appropriate for the characteristics of qualitative case studies, as the purpose of the research is not to represent the population but to conduct in-depth per-case analysis and cross-case analysis. In qualitative research, a limited number of subjects can make a meaningful contribution when the data collected is rich and analyzed reflectively and theoretically (Creswell, 2012; Miles et al., 2014). Thus, the two contrasting cases in this study are considered sufficient to analytically explain the role of cognitive flexibility in the context of algebraic misconceptions.

**Instruments**

The research instruments consisted of a written test and semi-structured interviews. The written test consisted of 12 algebraic root questions developed based on a study of radical misconceptions and strategy flexibility, covering: (1) diagnostic misconception questions (e.g., simplifying and assessing the equality of two root expressions), (2) strategy flexibility questions that require more than one solution method, and (3) adaptability questions with modified conditions. Students' answers were assessed using two rubrics, namely a mathematical correctness rubric (score 0–3) and a cognitive flexibility rubric (score 0–6) covering the aspects of variety, shifting, and justification/reflection. Semi-structured interviews were used as stimulated recall to explore the reasons for strategy selection, strategy shifting, and students' reflections on their answers, such as "*Why did you choose that strategy?*" and "*Try to show another way. Why is that way different?*".

The validity of the instrument was ensured through content validity, involving two mathematics education experts who examined the suitability of the questions and assessment rubrics with the research objectives and constructs being measured. The instrument was revised based on expert input until conceptual agreement was reached. The reliability of the assessment was maintained through scoring consistency, whereby scoring was carried out repeatedly and reconfirmed on some of the data to ensure the stability of the rubric interpretation. This approach is in line with qualitative research practices that emphasize trustworthiness through clear criteria, consistent analysis, and traceable assessment processes (Creswell, 2012; Miles et al., 2014).

**Procedure/Data Collection**

Data collection was conducted in three steps: (1) administering a written test to all students in the class; (2) initial scoring of all answers using both rubrics and selecting NH–YH as contrast cases; (3) conducting in-depth individual interviews with both subjects, referring to their answer sheets. All interviews were recorded and transcribed, and student identities were protected through the use of pseudonyms.

**Data Analysis**

Data analysis was conducted in two stages, namely descriptive quantitative analysis and qualitative analysis. Quantitatively, the mathematical correctness and cognitive flexibility scores for each item were summarized in a table (score per item, total, and average) to capture

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the initial profile of each subject. These quantitative results were used as an initial framework (analytic frame) for interpreting the qualitative data and selecting the focus of analysis on items that revealed misconceptions and variations in strategy.

Qualitatively, the analysis was conducted through a process of gradual thematic coding of written answers and interview transcripts. The first stage was open coding, which involved assigning initial codes to data units representing misconceptions, solution strategies, strategy shifts, and forms of student justification. The second stage was axial coding, which involved grouping these codes into categories that were in line with the research analysis framework, namely types of misconceptions and aspects of cognitive flexibility (variety, shifting, and justification/reflection). The analysis was conducted within each case for each subject (NH and YH), followed by a cross-case analysis to compare reasoning patterns and explain how differences in cognitive flexibility relate to students' tendencies to maintain or revise misconceptions in algebra tasks (Creswell, 2012; Miles et al., 2014).

## Research Results

Descriptive analysis of student performance on algebra tasks provides an initial overview of their misconception profiles and levels of cognitive flexibility. Mathematical correctness was assessed on a scale of 0–3 for each of the 12 items, with higher scores indicating more accurate answers based on correct concepts. As summarized in Table 1, subject NH obtained a total of 33 out of a maximum score of 36 (average 2.75), while YH obtained 29 (average 2.42). Both subjects showed a relatively high level of correctness, but with differences in the stability and strength of their conceptual explanations. On the diagnostic items targeting misconceptions about root forms (Items 1–6), NH consistently produced correct simplifications and explanations, while YH sometimes arrived at the correct final answer but used less appropriate reasoning, for example, emphasizing the "coefficient inside the root" as the basis for simplification.

**Table 1.** Mathematical correctness scores for each question number (1–12)

| No. Quest. | Instruments | Score NH (0-3) | Score YH (0-3) | Brief Notes (optional) |
|---|---|---|---|---|
| 1 | I - Misconception Diagnostics | 2 | 2 | Both are correct; the forms cannot be added together, and the term "coefficient in the root" is inaccurate |
| 2 | I - Misconception Diagnostics | 3 | 1 | NH is correct, emphasizing the difference between variables (x and y). YH is correct in its conclusion, but the reason "different coefficients" is inaccurate |
| 3 | I - Misconception Diagnostics | 3 | 3 | Simplifying $\sqrt{2x} + \sqrt{8x}$ to $3\sqrt{2x}$ is considered correct for both |
| 4 | I - Misconception Diagnostics | 3 | 1 | NH simplifies $\sqrt{20y}$ and gives the correct conclusion. YH only states that it cannot be added without supporting analysis |
| 5 | I - Misconception Diagnostics | 3 | 3 | Both are correct in stating that the form $\sqrt{ax} + \sqrt{ay}$ (or $\sqrt{ax} + \sqrt{by}$) is not the same as $\sqrt{a(x + y)}$ |
| 6 | I - Misconception Diagnostics | 3 | 3 | $2\sqrt{3x} + 3\sqrt{12x}$ is correctly simplified to $8\sqrt{3x}$ by both |
| 7 | II - Strategy Flexibility Task | 2 | 3 | NH only writes one correct strategy. YH writes two equivalent ways to get to $5\sqrt{2x}$ |

| | | | | |
|---|---|---|---|---|
| 8 | II - Strategy Flexibility Task | 2 | 3 | NH wrote two correct strategies, but there was a typo (5 + 2) √2x. YH wrote two clear and correct strategies |
| 9 | II - Strategy Flexibility Task | 3 | 2 | NH wrote two correct strategies for √45p + √5q. YH wrote only one correct strategy without other variations |
| 10 | III - Adaptability (condition variation) | 3 | 3 | Both found a = 3 and showed that the value of a remained the same when x was replaced with 4x |
| 11 | III - Adaptability (condition variation) | 3 | 2 | NH used consistent case analysis and examples. YH's idea was correct, but there was an incorrect substitution (1 + 4 was written as 2 + 4) |
| 12 | III - Adaptability (condition variation) | 3 | 3 | Both simplify the form correctly and conclude that the initial statement is false |
| | Total | 33 | 29 | Maximum score 36 (12 questions × score 3) |
| | Average | 2,75 | 2,42 | verage per question (Total divided by 12), rounded to two decimal places |

Cognitive flexibility was assessed on strategy and adaptability tasks (Items 7–12) using a three-dimensional rubric covering Variety, Shifting, and Justification (each scored 0–2; total 0–6 per item). As shown in Table 2, NH obtained a total flexibility score of 19 out of 36 (average 3.17), while YH obtained 22 (average 3.67). Both of these average scores fall into the moderate flexibility category, indicating that students are able to generate alternative strategies and adjust their reasoning to a certain extent, but do not yet consistently display highly flexible and reflective use of strategies across all items. This quantitative pattern was then enriched with interview data, which showed how the differences in accuracy and flexibility scores were consistent with how each subject explained their reasons for choosing a strategy and how they checked their answers.

**Table 2.** Cognitive Flexibility Scores for each Question (Tasks II & III)

| No. Quest. | Task Section | NH Var | NH Shift | NH Just | NH Score (0-6) | YH Var | YH Shift | YH Just | YH Score (0-6) |
|---|---|---|---|---|---|---|---|---|---|
| 7 | II - Strategy Flexibility | 0 | 0 | 1 | 1 | 1 | 2 | 1 | 4 |
| 8 | II - Strategy Flexibility | 1 | 2 | 1 | 4 | 1 | 2 | 1 | 4 |
| 9 | II - Strategy Flexibility | 1 | 2 | 2 | 5 | 0 | 0 | 1 | 1 |
| 10 | III - Adaptability (variation in conditions) | 0 | 1 | 2 | 3 | 0 | 1 | 2 | 3 |
| 11 | III - Adaptability (variation in conditions) | 1 | 2 | 2 | 5 | 1 | 2 | 2 | 5 |
| 12 | III - Adaptability (variation in conditions) | 0 | 0 | 1 | 1 | 1 | 2 | 2 | 5 |
| | Total | | | | 19 | | | | 22 |
| | Average | | | | 3.17 | | | | 3.67 |

An integrated reading of truth scores and flexibility scores indicates that the relationship between cognitive flexibility and the ability to resolve misconceptions is nuanced and non-linear. NH, who obtained the highest truth score, was not the subject with the highest flexibility score; NH's profile is more accurately described as conceptually stable with moderate flexibility. After choosing a valid strategy, NH tends to apply it efficiently and consistently,

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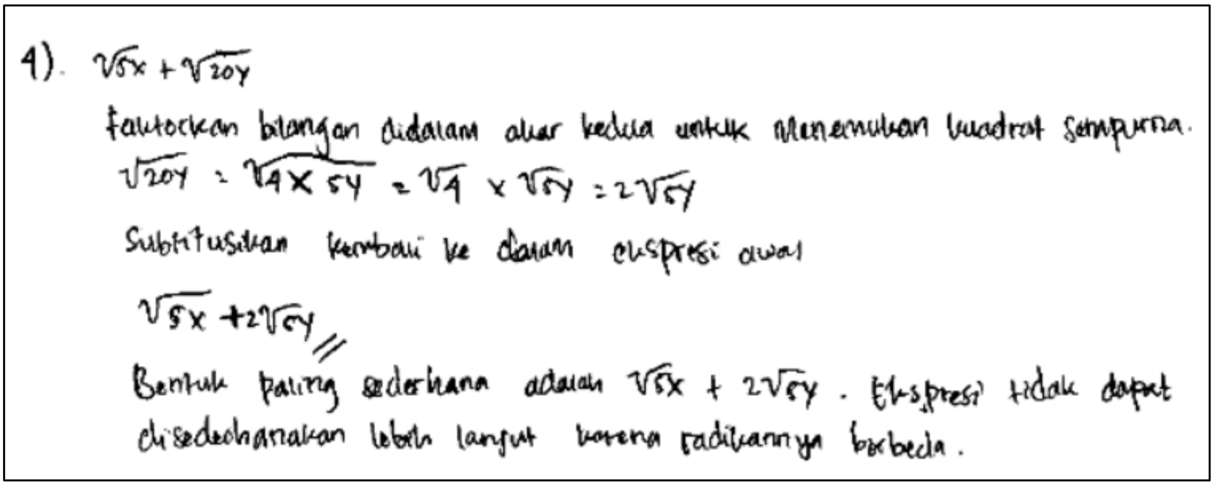
with relatively limited explicit exploration of alternative strategies. This is consistent with NH's statement in the interview that he "*chooses to simplify the radicand first, factor the numbers inside the root to find perfect squares, then check for radicand equality,*" and he considers this sequence to be the "*safest strategy that can be explained step by step.*".

In contrast, YH showed slightly higher flexibility in terms of scores, especially in the aspects of Variety and Shifting, but this did not necessarily result in better overall correctness. Some of YH's flexible steps were based on fragile conceptual understanding. In an interview, YH revealed that he relied heavily on procedurally memorized rules, such as "*roots cannot be added or subtracted*," and admitted to feeling uncertain when asked to factor numbers inside roots or change their form. He stated that he trusted the simple first step of "*just write down the form as it is, then conclude that it cannot be simplified*" rather than trying other strategies that required deeper conceptual reasoning.

From the perspective of the relationship between the two variables, these two profiles suggest that cognitive flexibility is an important resource but is not sufficient to guarantee the resolution of misconceptions in algebra. The ability to generate and shift between multiple strategies does open up opportunities for students to revisit their initial interpretations of problems, but these opportunities only lead to conceptual change when supported by a solid knowledge structure. In this data, higher truth seems to be related to the stability of radical concepts and awareness of the domain (as described by NH when he emphasized checking the value of x and the non-negativity condition in the root), while higher flexibility without a corresponding conceptual foundation (as in YH) still leaves unresolved misconceptions.

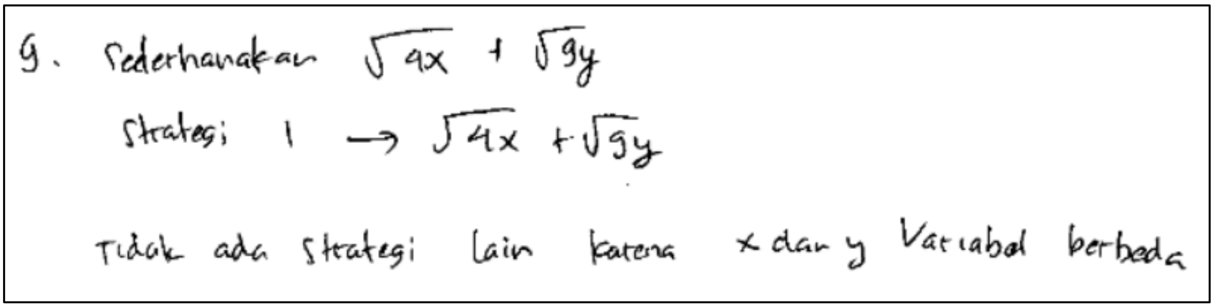



(Source: Research Data)

**Figure 1.** One of NH's Answers

To deepen the quantitative pattern, a qualitative analysis was conducted on the written answers and reconstructed interviews of the two subjects. The focus of the analysis was how they chose, combined, and revised strategies when faced with potential misconceptions in root forms. In the diagnostic task, YH concluded several times that the two expressions "*could not be simplified further*" simply because the numbers in the roots were different, in line with his statement in the interview: "*I was taught from the beginning that root forms cannot be combined in addition and subtraction... So I just stick to the rule that I remember: 'roots cannot be added or subtracted'.*" This statement indicates an overly rigid generalization of the rule and reinforces YH's misconception profile regarding root form operations. In contrast, NH's written answer (Figure 1) shows a more structured procedure, which is consistent with NH's explanation that "*the most basic step for root forms is to simplify the radicand first... then I see* *whether the radicands are the same or different... that strategy feels safest because it is in line with what I was taught and I can explain it step by step.*".

Evidence of cognitive flexibility is evident when students are asked to show other ways. In the interview, NH not only relied on algebraic manipulation but also utilized numerical case analysis as a second strategy: "*I substitute first* $(x = 1)$ *or* $(x = 4)$*, then I calculate the numerical value... I use this second strategy as a 'double-check' of the algebraic strategy.*" He even emphasized that he trusted steps most when they could be explained and retested: "*I trust steps most when I can explain the reasoning and retest them in other ways, not just follow the formula by rote.*" These quotes show that NH's flexibility was not merely about producing multiple strategies, but also accompanied by reflection on why certain strategies were considered more reliable. This profile is evident in the adaptability task, where NH consistently checks the equivalence of forms by substituting different values of $x$ and considering domain conditions.




(Source: Research Data)

**Figure 2.** One of YH's Answers

Conversely, when asked to demonstrate alternative strategies, YH admitted to having "*some difficulty thinking of other ways, because I feel that the basic rules are already clear*" and only mentioned the possibility of writing down more detailed steps or calculating the decimal approach, while emphasizing that this method "*did not feel official*" to him. In another part of the interview, YH stated: *"I trust the steps most when I directly apply the rules I remember... In fact, if I have to factor numbers in the root or change the form, I feel more uncertain."* This shows that technically YH can name or imagine other strategies, but epistemically he still returns to memorized rules and is reluctant to evaluate or revise his ingrained schemas. YH's written answers on the algebra assignment (Figure 2) are consistent with this: the initial form is often retained without any systematic attempt to simplify or prove the equivalence of forms.

When read together with quantitative data, these qualitative narrative excerpts clarify how cognitive flexibility plays a role (and is sometimes inhibited) in the resolution of misconceptions. NH represents a student with a fairly robust radical conceptual schema, who utilizes flexibility through a combination of algebraic strategies and case analysis to test and reinforce understanding. YH represents a student with strong misconceptions about root operations, whose flexibility is limited by adherence to memorized procedural rules and a lack of confidence in strategies that require deeper understanding. These findings confirm that the role of cognitive flexibility in resolving misconceptions cannot be separated from the quality of the underlying conceptual understanding.

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**Discussion**

This section discusses the research findings with reference to two research questions, namely: (1) identifying the profile and characteristics of students' misconceptions on radical algebra material through a series of diagnostic tasks, and (2) describing the profile of students' cognitive flexibility in terms of variety, shifting, and justification/reflection in completing strategy tasks and adaptability tasks in the context of algebra. The discussion combines quantitative evidence (accuracy and flexibility scores) with qualitative evidence from written work and interviews, and relates them to the literature review on algebraic misconceptions and cognitive flexibility theory.

**Profile and Characteristics of Misconceptions in Algebraic Radicals**

The findings in the results section show that both subjects have relatively high levels of mathematical correctness, but still retain certain patterns of misconceptions, especially in root form operations. This pattern is consistent with various studies showing that students often experience difficulties when moving from ordinary arithmetic operations to operations on algebraic expressions, including root forms, causing them to generalize rules incorrectly (Vijaganita & Alifiani, 2024).

In subject YH, misconceptions were evident when he tended to adhere to the memorized rule that "roots cannot be added or subtracted," so that any expression of root form addition was considered "cannot be simplified further" simply because the numbers inside the roots were different. This pattern is in line with Özkan (2011) finding that many students do not view radical expressions as a single object determined by the "root degree–radicand" pair, but rather focus on the numbers/coefficients that appear inside the root symbol. Other studies in the Indonesian context also report that junior high and high school students often have misconceptions about the definition and operations of root forms, particularly in addition/subtraction and root form equality (Hamid, 2024; Hindi & HR, 2022; Setyaningtyas & Muksar, 2018). In contrast, subject NH showed a more conceptual profile. He consistently used the steps of simplifying radicals by factoring, finding perfect squares, then checking the equality of radicals before combining like terms. This strategy is in line with the principle of teaching root forms that emphasizes understanding the structure of radicals, not just symbolic manipulation (Özkan, 2011). The quality of NH's reasoning supports the findings Vijaganita & Alifiani (2024) that misconceptions in simplifying algebraic expressions often arise when students rely solely on old assimilation schemes (e.g., the usual addition rule) without accommodating the specific properties of the algebraic forms they encounter.

From the perspective of the first research question, these data show that misconceptions appear not only as "wrong answers," but also as patterns of reasoning that overgeneralize rules (case YH) or, conversely, as more organized conceptual schemas (case NH). This is in line with a systematic review that confirms that algebraic misconceptions are often persistent and bound to established thinking schemes, thus requiring interventions that explicitly target schema restructuring, rather than merely procedural exercises (Ridho & Juandi, 2023).

**Cognitive Flexibility Profile: Variety, Shifting, and Justification**

Regarding the second research question, the findings show that both subjects are in the moderate cognitive flexibility category, with YH slightly higher than NH in terms of total scores for variety, shifting, and justification. Theoretically, mathematical flexibility is defined as the ability to use various strategies, representations, and concepts flexibly, meaningfully, and innovatively (Hickendorff et al., 2022). In some literature, flexibility is viewed as an important outcome of mathematics learning, but it is also "capricious" and does not automatically emerge simply because of increased learning experience (Heinze et al., 2009).

In this study, NH demonstrated a pattern of selective flexibility. On some items of strategy and adaptability, he was able to use two different approaches, for example, switching from algebraic manipulation to numerical case analysis to test form equivalence. However, in general, he tended to choose one strategy that was considered the most efficient and then use it consistently. This reflects the profile of a student with moderate flexibility, but with relatively strong justification/reflection: he can explain why a particular strategy was chosen and how other strategies function as "checks".

YH, on the other hand, more often displays variety and shifting on the surface, for example, by mentioning the possibility of calculating in decimals or writing down more detailed steps. However, in terms of justification, he acknowledges that these alternative strategies do not feel "official" and he does not trust them as much. This pattern is in line with the findings of Coppersmith & Star (2022), who concluded that the relationship between strategy flexibility and accuracy is "complicated", students can exhibit flexible behavior procedurally, but this is not always followed by an increase in correctness or depth of understanding.

Thus, the cognitive flexibility profiles of the two subjects in this study reinforce the view that flexibility is not only a matter of "how many strategies one has," but also the extent to which students are able to provide reflective justification, evaluate, and choose the most appropriate strategy for a particular task context. This is in line with studies on mathematical flexibility that emphasize that meaningful flexibility must involve coordination between conceptual and procedural knowledge, not just surface variations in procedures (Coppersmith & Star, 2022; Laia et al., 2025).

When the two findings above are combined, it appears that the role of cognitive flexibility in resolving misconceptions is conditional. On the one hand, flexibility opens up opportunities for constructive cognitive conflict: when students compare two different strategies (e.g., algebraic manipulation versus numerical substitution), they may realize inconsistencies in their thinking and be motivated to revise their misconceptions. The cognitive conflict-based approach itself has been proven effective in reducing misconceptions about exponents and roots, for example through the use of tiered diagnostic tests and reflective discussions (Güveli et al., 2022; HR et al., 2023; Parwati & Suharta, 2020; Santosa et al., 2025; Winarso & Udin, 2023).

However, the data from the two cases in this study also show that flexibility without a strong conceptual foundation is not sufficient to guarantee the elimination of misconceptions. Although YH had a slightly higher flexibility score, misconceptions about root operations persisted because he relied heavily on memorized procedural rules and viewed alternative strategies as "unofficial." This pattern is consistent with the findings of a recent review of mathematical flexibility, which states that flexibility is the result of a complex learning process, and its relationship with performance is not always linear or simply positive (Hickendorff et al., 2022).

In contrast, NH demonstrates how flexibility can directly contribute to the resolution of misconceptions when combined with a relatively robust conceptual schema. By using a second strategy of numerical case analysis and domain checking, NH not only repeats the procedure but also tests the validity of the algebraic transformations he performs. This is in line with the *Cognitive Flexibility Theory* framework, which emphasizes the importance of viewing a concept from various representations and contexts so that the knowledge built is adaptive and not stuck in overly simplistic schemas (Secolsky & Denison, 2018). These findings provide evidence that the main role of cognitive flexibility in resolving misconceptions is not merely to

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generate multiple strategies, but to provide "cognitive space" for students to compare, test, and ultimately reconstruct their understanding. When flexibility is supported by a proper understanding of radical concepts, as in NH, students tend to use alternative strategies to confirm and refine their knowledge. However, when flexibility is based on a fragile understanding, as in YH, students may use additional strategies to reinforce their belief in their misconceptions rather than correct them.

### Pedagogical Implications and Limitations of the Study

Pedagogically, the results of this study indicate that efforts to develop cognitive flexibility should not be separated from explicit interventions on misconceptions. Mathematics teachers should not simply encourage students to "find other ways," but need to design tasks and classroom discussions that deliberately confront correct and incorrect strategies, ask students to compare solutions, and guide them to provide reflective justifications for their choice of strategy. This approach is consistent with research showing that comparing multiple solution strategies can enhance both procedural flexibility and conceptual understanding in algebra (Star & Rittle-Johnson, 2008).

Furthermore, recent literature reviews on flexibility in mathematics learning emphasize the importance of interactive and flexible classroom environments, where teachers adaptively adjust representations, questions, and forms of support (scaffolding) according to student needs. Munaji et al., (2025) in the context of root forms, this can be achieved through the use of various representations (symbolic, numerical, and visual), expression comparison tasks that require students to identify the conditions under which root forms can be combined, and the use of clinical interviews or reflective discussions to uncover the thinking schemes underlying misconceptions.

In terms of limitations, this study is a case study with two subjects, so the findings cannot be statistically generalized to a wider population. However, the depth of the written task data and interviews allows for a rich reconstruction of misconception profiles and cognitive flexibility, which can serve as a basis for further studies with larger samples and mixed quantitative-qualitative designs (Kristie & Star, 2010; Newton et al., 2019). Another limitation is the focus on the topic of radical algebra; flexibility and misconceptions on other algebra topics (e.g., quadratic equations or functions) may exhibit different dynamics and need to be studied specifically (Baybayon & Lapinid, 2024).

Nevertheless, these findings contribute conceptually to the understanding of the role of cognitive flexibility in resolving algebraic misconceptions. This study confirms that learning interventions targeting flexibility need to be designed in conjunction with efforts to strengthen conceptual understanding, for example through cognitive conflict strategies, solution comparison tasks, and digital conceptual mapping that highlights the relationships between algebraic concepts (Elhilal, 2025; Rüede et al., 2023).

## Conclusion

This study concludes that students' misconceptions about algebraic root operations are not solely caused by procedural errors, but are rooted in the way students construct and maintain the mathematical rules they believe in. Misconceptions tend to arise when rules are understood separately from the conceptual structure of root forms, causing students to rigidly generalize certain principles without considering the equivalence of forms and accompanying conditions. Conversely, a more structured conceptual understanding allows students to interpret root forms in a more meaningful and consistent manner.

The main findings of this study confirm that cognitive flexibility plays an important role in responding to misconceptions, but that role is conditional. Cognitive flexibility does not automatically work to correct misconceptions; its effectiveness only becomes apparent when students have a sufficiently strong conceptual foundation to assess and compare the strategies used. Flexibility accompanied by reflection and justification allows students to use alternative strategies as tools to test and revise their understanding, whereas flexibility based on memorized rules tends to only produce procedural variations without meaningful conceptual change.

Thus, the main contribution of this study lies in confirming that the relationship between cognitive flexibility and the resolution of algebraic misconceptions is not linear, but rather mediated by the quality of students' conceptual understanding. Practically, the implication of these findings for mathematics teachers is the need to design root form learning that not only encourages the use of various strategies but also guides students to compare, justify, and reflect on these strategies conceptually, so that cognitive flexibility truly functions as a means of changing understanding, not merely a variation in calculation methods.

**Conflict of Interest**

The author declares no conflict of interest.

**Authors' Contributions**

A.N.AM.H played a role in conducting this research, including data collection, data analysis, interpretation of results, and manuscript preparation. Meanwhile, U.M provided guidance in the implementation of the study, ensured methodological suitability, and approved the final version of the manuscript. All authors have read and approved the final version of this article. The percentage contribution to the conception, writing, and revision of this article is as follows: A.N.AM.H: 60% and U.M:40%.

**Data Availability Statement**

The authors state that data sharing is not possible, as no new data were created or analyzed in this study

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