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Determinants of Students' Mathematics Achievement: The Roles of Critical Thinking, Self-Efficacy, and Metacognitive Awareness Through Achievement Motivation

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ABSTRACT

This study examines whether students' internal factors, namely critical thinking ability, self-efficacy, and metacognitive awareness, contribute to mathematics achievement directly and indirectly through achievement motivation. This study therefore aimed to analyze the direct and indirect effects of critical thinking ability, self-efficacy, and metacognitive awareness on mathematics achievement through achievement motivation among 121 eighth-grade students. A quantitative ex post facto design was employed, and the data were analyzed using path analysis to investigate the proposed causal relationships. Descriptive findings showed that students' critical thinking ability was at a moderate level ($M = 75.6$), self-efficacy was high ($M = 61.5$), metacognitive awareness was high ($M = 89.3$), and achievement motivation was moderate ($M = 58.4$). However, mathematics achievement was also only at a moderate level ($M = 59.6$). The results revealed that critical thinking ability had a significant direct effect on mathematics achievement ($\beta = 0.1470$, $p = 0.045$). By contrast, self-efficacy ($\beta = -0.0906$, $p = 0.785$), metacognitive awareness ($\beta = 0.0927$, $p = 0.561$), and achievement motivation ($\beta = 0.3753$, $p = 0.141$) did not show significant direct effects on mathematics achievement. In the first path model, only self-efficacy ($\beta = 0.9256$, $p < 0.001$) and metacognitive awareness ($\beta = 0.1274$, $p = 0.027$) significantly predicted achievement motivation. Overall, the model explained only 3.07% of the variance in mathematics achievement ($R^2 = 0.0307$). These findings indicate that critical thinking ability is the only variable in the model that significantly predicts mathematics achievement, while other factors outside the model may have a more substantial role.



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Introduction

Improving the quality of education remains a central concern in efforts to prepare students for increasingly complex social and intellectual demands. Education is not merely a process of transmitting knowledge, but a deliberate and systematic effort to create learning environments in which students can develop their intellectual, social, emotional, and moral potential (Gulkilik et al., 2020; Weiland et al., 2020; Yurekli, Stein, et al., 2020). In this context, the quality of classroom learning becomes a decisive factor, because educational success depends not only on what is taught, but also on how students engage with ideas, tasks, and learning challenges (Callingham & Siemon, 2021; Hackenberg et al., 2021). Mathematics education holds a particularly important position within this agenda, as it is expected to foster logical, analytical, critical, and creative thinking that supports both academic development and everyday problem solving (Geretschläger & Donner, 2022; Vidic et al., 2022).

Despite its importance, mathematics achievement remains a persistent challenge in Indonesia. Mathematics is a compulsory subject across all levels of formal education and serves as a key indicator of instructional effectiveness (H. S. Lee et al., 2023). However, empirical evidence continues to show that Indonesian students' performance in mathematics is relatively low. The results of PISA 2022 indicate that Indonesia's average mathematics score declined to 366, remaining far below the OECD average (Safrudiannur & Rott, 2019). Although Indonesia's ranking slightly improved compared with the previous cycle, only a small proportion of students achieved the minimum level of mathematical proficiency (Cromley et al., 2017). These findings suggest that many students still struggle to develop adequate conceptual understanding and problem-solving competence in mathematics. This condition calls for closer attention to the factors that shape students' mathematics achievement.

Academic achievement is influenced by a combination of external and internal factors. External factors include instructional methods, classroom climate, and family support, whereas internal factors are more directly associated with students' cognitive and affective functioning (Ma'rufi et al., 2020; Ogrodzka-Mazur et al., 2017; Şahin et al., 2020). Among these internal factors, critical thinking, self-efficacy, metacognitive awareness, and achievement motivation are frequently identified as important predictors of successful learning (Y. Lee et al., 2019). Internal factors are especially important because they influence how students interpret academic demands, regulate their learning processes, and persist when facing difficulty. Understanding the role of these variables is therefore essential for designing learning environments that are more responsive to students' needs and more effective in improving mathematics achievement.

Self-efficacy is one of the internal variables most consistently associated with academic success. Regier & Savic (2020) defines self-efficacy as individuals' beliefs in their capability to organize and execute the actions required to attain desired outcomes. In mathematics learning, self-efficacy influences how students approach difficult tasks, respond to failure, and sustain effort during problem solving. Students with higher self-efficacy are more likely to persist, apply effective strategies, and participate actively in learning activities, whereas those with lower self-efficacy tend to avoid challenge and disengage more easily. Previous studies have reported a positive relationship between self-efficacy and mathematics achievement (Hamidy et al., 2023; Street et al., 2022; Yurekli, et al., 2020), suggesting that students' beliefs about their capability may meaningfully shape their learning outcomes.

Metacognitive awareness is another important factor in mathematics learning. Hartmann et al., (2024) conceptualized metacognition as individuals' awareness and regulation of their own thinking processes, including planning, monitoring, and evaluating learning strategies. In mathematics, metacognitive awareness helps students assess whether they understand a

problem, select appropriate procedures, and reconsider ineffective strategies. Students with stronger metacognitive awareness are generally better able to regulate their learning and solve complex tasks efficiently. Prior research has shown that metacognition contributes positively to academic performance, particularly in domains that require higher-order thinking (Hartmann et al., 2024; Hoang, 2020). In mathematics, metacognitive regulation is especially relevant because successful problem solving often depends not only on what students know, but also on how well they manage their thinking while working through unfamiliar situations.

Critical thinking also occupies a central role in mathematics learning. Kuntze et al., (2017) defines critical thinking as reflective and rational thinking directed toward deciding what to believe or do. In mathematics, this ability enables students to evaluate information, examine assumptions, justify procedures, and draw conclusions based on evidence and logical reasoning. Students with stronger critical thinking skills are more likely to develop deeper conceptual understanding and to solve problems more effectively than those who rely only on routine procedures. Empirical studies have shown that critical thinking is positively associated with mathematics achievement (Kuntze et al., 2017; Lehmann, 2023; Zanden et al., 2020). This suggests that mathematics learning outcomes may improve when students are encouraged to reason carefully, question information, and construct well-supported conclusions.

In addition to these cognitive and self-regulatory variables, achievement motivation may play an important mediating role. Achievement motivation refers to the drive to attain success, meet standards of excellence, and persist in academic tasks (Khozaei et al., 2022). Students with stronger achievement motivation tend to show greater effort, persistence, and willingness to face academic challenges. In contrast, students with weaker motivation are more likely to avoid demanding tasks and disengage when difficulties arise. From a theoretical perspective, achievement motivation may strengthen the effect of critical thinking, self-efficacy, and metacognitive awareness on mathematics achievement, because these internal resources may be translated into performance more effectively when students are motivated to achieve. This makes achievement motivation a relevant mediating variable in explaining how internal factors are linked to academic outcomes.

Although previous studies have demonstrated positive associations between critical thinking, self-efficacy, and mathematics achievement (Regier & Savic, 2020; Street et al., 2022), important gaps remain. First, most prior studies have examined these variables separately rather than within an integrated explanatory model. Second, limited attention has been given to the simultaneous roles of critical thinking, self-efficacy, and metacognitive awareness in predicting mathematics achievement through achievement motivation. Third, evidence at the junior secondary school level remains relatively scarce, even though this stage is crucial for the development of students' mathematical reasoning and learning dispositions. As a result, the direct and indirect relationships among these internal factors have not yet been sufficiently clarified.

This study addresses these gaps by examining critical thinking, self-efficacy, and metacognitive awareness simultaneously within a single causal model, with achievement motivation positioned as a mediating variable. The study focuses on eighth-grade students in Tana Lili District, a context that has received limited empirical attention in the literature. The contribution of this study lies in providing a more integrated understanding of how students' internal cognitive and motivational factors interact in shaping mathematics achievement. Such an approach is expected to extend the current literature beyond isolated bivariate relationships and offer a more comprehensive account of the mechanisms underlying mathematics learning outcomes.

Accordingly, this study aims to analyze the direct and indirect effects of critical thinking, self-efficacy, and metacognitive awareness on mathematics achievement through achievement

motivation among eighth-grade junior secondary school students in Tana Lili District. The findings are expected to contribute theoretically to the literature on internal determinants of mathematics achievement and practically to support teachers and schools in designing instructional strategies that strengthen both students' cognitive capacities and their motivational engagement in mathematics learning.

Method

Design

This study employed a quantitative ex post facto design to examine the causal relationships among critical thinking ability, self-efficacy, metacognitive awareness, achievement motivation, and students' mathematics achievement. Ex post facto research focuses on analyzing variables and conditions that have already occurred without manipulation by the researcher. This design was considered appropriate because the study aimed to investigate both the direct and indirect effects among the variables within a naturally occurring educational context.

Population and Sample

The population of this study consisted of all junior secondary school students in Tana Lili District during the 2025/2026 academic year. The sample was selected using cluster random sampling to obtain a representative sample of the population. In the first stage, students were grouped into three grade levels, namely Grades VII, VIII, and IX. In the second stage, one grade level was randomly selected, and Grade VIII was chosen as the study sample. After the target grade had been determined, the sample size was calculated. The total population of Grade VIII students across all junior secondary schools in Tana Lili District was approximately 350 students. These 121 students were then selected proportionally from each school with Grade VIII classes to ensure balanced representation. As a result, the sample was distributed across all junior secondary schools in Tana Lili District, which improved the representativeness and generalizability of the findings.

Instruments

Data were collected using five instruments: a self-efficacy questionnaire, a metacognitive awareness questionnaire, an achievement motivation questionnaire, a critical thinking test, and a mathematics achievement test. The questionnaire instruments employed a four-point Likert scale with both favorable and unfavorable items to reduce response bias. The self-efficacy questionnaire consisted of 20 items, the metacognitive awareness questionnaire 30 items, and the achievement motivation questionnaire 20 items, while critical thinking was measured using three essay items and mathematics achievement was assessed through an essay-based test. Prior to the main study, all instruments were evaluated by expert validators to establish content and construct validity and were subsequently piloted with 63 students outside the main sample. The expert validation results showed that the instruments had very high validity, with coefficients ranging from 0.833 to 0.854. Reliability was examined using Cronbach's alpha in SPSS version 21, and all instruments met the acceptable reliability threshold of 0.60. These results indicate that the instruments were appropriate for use in the study.

Table 1. Summary of Instrument Validity and Reliability

Instrument	Number of Items	Validation Coefficient	Validity Category	Cronbach's Alpha	Reliability Category
Self-efficacy questionnaire	20	0.833	Very high	0.849	Reliable
Metacognitive awareness questionnaire	30	0.833	Very high	0.820	Reliable
Achievement motivation questionnaire	20	0.854	Very high	0.852	Reliable
Critical thinking test	3	Validated by experts	Valid	0.654	Reliable
Mathematics achievement test	Essay test	0.854	Very high	-	-

Procedure

Data were collected directly by the researcher from the selected sample in stages according to the schedule agreed upon with each school. The instruments included a self-efficacy questionnaire, a metacognitive awareness questionnaire, an achievement motivation questionnaire, a critical thinking test, and a mathematics achievement test. Data on self-efficacy, metacognitive awareness, and achievement motivation were obtained through questionnaires, whereas data on critical thinking and mathematics achievement were collected through essay-based tests. The questionnaires were scored based on students' selected responses, and the test results were used to represent students' critical thinking ability and mathematics achievement.

Analysis

The collected data were analyzed using descriptive and inferential statistics. Descriptive statistics were used to summarize the data without making generalizations and included the mean, median, variance, standard deviation, minimum score, and maximum score. These statistics were intended to provide a clear overview of students' critical thinking, self-efficacy, metacognitive awareness, achievement motivation, and mathematics achievement. Inferential statistics were used to test the proposed hypotheses. Prior to hypothesis testing, several assumption tests were conducted, including normality, multicollinearity, linearity, homoscedasticity, and outlier detection. Normality was tested using the Kolmogorov-Smirnov test, with data considered normally distributed when $p > .05$. Multicollinearity was assessed using the Variance Inflation Factor (VIF), and the independent variables were considered free from multicollinearity when $VIF < 5$. Linearity was examined to determine whether the relationships between variables were linear, with linearity assumed when the significance value for deviation from linearity was greater than .05. Hypothesis testing was conducted using path analysis in SPSS version 21 to estimate both the direct and indirect effects among the variables. Path analysis was employed to explain the structural relationships among the exogenous variables, the mediating variable, and the endogenous variable. In this study, X1 represented critical thinking ability, X2 self-efficacy, X3 metacognitive awareness, X4 achievement motivation, and Y mathematics achievement. The hypotheses tested in this study were as follows:

1. Critical thinking ability has a direct effect on mathematics achievement.
2. Self-efficacy has a direct effect on mathematics achievement.
3. Metacognitive awareness has a direct effect on mathematics achievement.
4. Achievement motivation has a direct effect on mathematics achievement.
5. Critical thinking ability has a direct effect on achievement motivation.

6. Self-efficacy has a direct effect on achievement motivation.
7. Metacognitive awareness has a direct effect on achievement motivation.
8. Critical thinking ability has an indirect effect on mathematics achievement through achievement motivation.
9. Self-efficacy has an indirect effect on mathematics achievement through achievement motivation.
10. Metacognitive awareness has an indirect effect on mathematics achievement through achievement motivation.

Research Findings

Descriptive Results

Table 2 presents the descriptive statistics for all study variables. Among the five variables, students reported relatively high levels of self-efficacy ($M = 61.5$) and metacognitive awareness ($M = 89.3$), while critical thinking was moderate ($M = 75.6$). Achievement motivation was also at a moderate level ($M = 58.4$), and mathematics achievement remained moderate ($M = 59.6$). These results suggest that favorable internal characteristics were not fully reflected in students' mathematics achievement.

Table 2. Descriptive Statistics of the Study Variables

Variable	N	Mean	Median	SD	Minimum	Maximum
Critical thinking	121	75.6	67	22.3	33	100
Self-efficacy	121	61.5	60	6.91	46	80
Metacognitive awareness	121	89.3	86	10.2	73	117
Achievement motivation	121	58.4	55	9.27	47	82
Mathematics achievement	121	59.6	60	17.6	10	100

The categorical distributions are shown in Table 3. Critical thinking was mostly in the moderate category (48.76%), self-efficacy in the high category (76.86%), metacognitive awareness in the high category (46.28%), and achievement motivation in the moderate category (77.69%). Mathematics achievement was predominantly moderate (39.67%), followed by low (28.10%). Overall, the descriptive results indicate that students tended to report positive internal dispositions, but their mathematics achievement remained modest.

Table 3. Frequency Distribution of the Study Variables

Variable	Category	Frequency	Percentage (%)
Critical thinking	Very high	47	38.84
	High	0	0.00
	Moderate	59	48.76
	Low	0	0.00
	Very low	15	12.40
Self-efficacy	Very high	1	0.83
	High	93	76.86
	Moderate	27	22.31
	Low	0	0.00
	Very low	0	0.00
Metacognitive awareness	Very high	19	15.70
	High	56	46.28
	Moderate	46	38.02
	Low	0	0.00
	Very low	0	0.00

Achievement motivation	Very high	2	1.65
	High	23	19.01
	Moderate	94	77.69
	Low	2	1.65
	Very low	0	0.00
Mathematics achievement	Very high	5	4.13
	High	21	17.36
	Moderate	48	39.67
	Low	34	28.10
	Very low	13	10.74

Assumption Testing

Prior to path analysis, assumption tests were performed. As shown in Table 4, all normality tests produced p values greater than .05, indicating that the data were normally distributed.

Table 4. Normality Test Results

Test	Statistic	p
Shapiro-Wilk	0.990	0.495
Kolmogorov-Smirnov	0.0639	0.706
Anderson-Darling	0.336	0.504

The multicollinearity test results in Table 5 show that all VIF values were below 5 and all tolerance values exceeded 0.10, indicating no serious multicollinearity among the predictors.

Table 5. Multicollinearity Test Results

Variable	VIF	Tolerance
Achievement motivation	1.06	0.941
Metacognitive awareness	2.25	0.444
Self-efficacy	1.07	0.937
Critical thinking	2.14	0.467

The linearity test results are presented in Table 6. Because all significance values were greater than .05, the relationships among the variables were considered linear.

Table 6. Linearity Test Results

Relationship Between Variables	p
Critical thinking × mathematics achievement	0.228
Self-efficacy × mathematics achievement	0.155
Metacognitive awareness × mathematics achievement	0.109
Achievement motivation × mathematics achievement	0.229
Critical thinking × achievement motivation	0.237
Self-efficacy × achievement motivation	0.728
Metacognitive awareness × achievement motivation	0.247

Path Analysis Results

Path Model I examined the effects of critical thinking, self-efficacy, and metacognitive awareness on achievement motivation. As shown in Table 7, self-efficacy significantly predicted achievement motivation ($\beta = 0.6900$, $p < .001$), and metacognitive awareness also had a significant positive effect ($\beta = 0.1399$, $p = .027$). In contrast, critical thinking did not significantly predict achievement motivation ($\beta = 0.0820$, $p = .196$).

Table 7. Path Coefficients for Model I Predicting Achievement Motivation

Predictor	Estimate	SE	t	p	Standardized Estimate	95% CI Lower	95% CI Upper
Intercept	-12.5045	6.7235	-1.86	0.065			
Critical thinking	0.0341	0.0262	1.30	0.196	0.0820	-0.0429	0.207
Self-efficacy	0.9256	0.0853	10.85	<.001	0.6900	0.5640	0.816
Metacognitive awareness	0.1274	0.0568	2.25	0.027	0.1399	0.0165	0.263

The model fit indices for Path Model I are reported in Table 8. The predictors explained 55.6% of the variance in achievement motivation, indicating substantial explanatory power.

Table 8. Model Fit for Path Model I

Model	R	R ²	Adjusted R ²
1	0.746	0.556	0.545

The structural equation for Path Model I was:

$$X_4 = 0.03X_1 + 0.93X_2 + 0.13X_3 + 0.66$$

Path Model II examined the effects of critical thinking, self-efficacy, metacognitive awareness, and achievement motivation on mathematics achievement. As shown in Table 9, only critical thinking had a significant direct effect on mathematics achievement ($\beta = 0.1856$, $p = .045$). The effects of achievement motivation ($p = .141$), metacognitive awareness ($p = .561$), and self-efficacy ($p = .785$) were not significant.

Table 9. Path Coefficients for Model II Predicting Mathematics Achievement

Predictor	Estimate	SE	t	p	Standardized Estimate	95% CI Lower	95% CI Upper
Intercept	23.8818	18.7017	1.277	0.204			
Critical thinking	0.1470	0.0724	2.029	0.045	0.1856	0.00445	0.367
Achievement motivation	0.3753	0.2534	1.481	0.141	0.1972	-0.06651	0.461
Metacognitive awareness	0.0927	0.1589	0.584	0.561	0.0535	-0.12802	0.235
Self-efficacy	-0.0906	0.3313	-0.273	0.785	-0.0355	-0.29237	0.221

As shown in Table 10, Path Model II explained only 8.78% of the variance in mathematics achievement, indicating limited predictive power.

Table 10. Model Fit for Path Model II

Model	R	R ²	Adjusted R ²
1	0.296	0.0878	0.0563

The structural equation for Path Model II was:

$$Y = 0.15X_1 - 0.09X_2 + 0.09X_3 + 0.38X_4 + 0.955$$

Hypothesis Testing

The direct-effect results are summarized in Table 11. The findings show that critical thinking significantly predicted mathematics achievement, while self-efficacy, metacognitive awareness, and achievement motivation did not. In addition, self-efficacy and metacognitive awareness significantly predicted achievement motivation, whereas critical thinking did not.

Table 11. Summary of Direct Effects

Hypothesis	Path	β	p	Decision
H1	Critical thinking → mathematics achievement	0.1470	0.045	Supported
H2	Self-efficacy → mathematics achievement	-0.0906	0.785	Not supported
H3	Metacognitive awareness → mathematics achievement	0.0927	0.561	Not supported
H4	Achievement motivation → mathematics achievement	0.3753	0.141	Not supported
H5	Critical thinking → achievement motivation	0.0341	0.196	Not supported
H6	Self-efficacy → achievement motivation	0.9256	< .001	Supported
H7	Metacognitive awareness → achievement motivation	0.1274	0.027	Supported

The indirect effects through achievement motivation are reported in Table 12. The indirect effect of critical thinking on mathematics achievement was very small (0.0114), indicating that its direct effect was stronger than its mediated effect. The indirect effect of self-efficacy was larger (0.3534), but because the path from achievement motivation to mathematics achievement was not significant, this mediated effect should be interpreted cautiously. The indirect effect of metacognitive awareness was also relatively small (0.0494).

Table 12. Summary of Direct and Indirect Effects Through Achievement Motivation

Exogenous Variable	Direct Effect on Mathematics Achievement (β)	p		Indirect Effect Through Achievement Motivation
Critical thinking	0.15	0.045	0.0114	
Self-efficacy	-0.09	0.785	0.3534	
Metacognitive awareness	0.09	0.561	0.0494	
Achievement motivation	0.375	0.141	-	

Overall, the results indicate that critical thinking was the only variable that significantly predicted mathematics achievement directly. Although self-efficacy and metacognitive awareness significantly predicted achievement motivation, the mediating role of achievement motivation was not supported because it did not significantly predict mathematics achievement.

Discussion

This study examined the relationships among critical thinking, self-efficacy, metacognitive awareness, achievement motivation, and mathematics achievement among eighth-grade students. The descriptive findings showed that self-efficacy and metacognitive awareness were relatively high, whereas critical thinking, achievement motivation, and mathematics achievement were only moderate. This pattern suggests that students' positive internal dispositions were not fully translated into stronger mathematics performance. In practical terms, confidence in one's ability and awareness of one's own learning processes may not be sufficient to produce higher achievement unless they are accompanied by strong analytical skills and effective engagement with mathematical tasks (Hamidy et al., 2023; Street

et al., 2022; Turan & Koç, 2018). This finding points to an important distinction between perceived learning resources and actual academic performance in mathematics.

The main finding of this study is that critical thinking was the only variable that significantly predicted mathematics achievement. This result highlights the central role of higher-order thinking in mathematics learning. Mathematics achievement is not merely a function of confidence or motivation, but also depends on students' ability to examine information, evaluate procedures, detect errors, and justify conclusions logically (Kinneer et al., 2024; Ødegaard et al., 2024; Paoletti et al., 2024). This finding is consistent with previous studies emphasizing that critical thinking is closely linked to problem solving, mathematical reasoning, and conceptual understanding. In this respect, the present study reinforces the argument that mathematics instruction should move beyond routine procedures and provide students with regular opportunities to engage in reflective and analytical reasoning (Fonger, 2019). The significant contribution of critical thinking also suggests that mathematics achievement may be more strongly shaped by cognitive processing quality than by self-perceptions alone.

By contrast, self-efficacy did not have a significant direct effect on mathematics achievement, even though it significantly predicted achievement motivation. This result partly diverges from studies that report a positive contribution of self-efficacy to academic performance. A plausible explanation is that students' reported confidence may not have matched the actual cognitive demands of the mathematics tasks. Students may have believed that they were capable, yet this belief may not have been strong enough to support successful performance on tasks requiring deep conceptual understanding and non-routine reasoning (Kaiser et al., 2025). Another explanation is that self-efficacy may operate more strongly as a motivational condition than as a direct academic predictor in this context. The finding that self-efficacy significantly predicted achievement motivation supports this interpretation. Thus, self-efficacy remains important, but its influence in this study appears to be more psychological than directly performance-based.

A similar pattern emerged for metacognitive awareness. Although metacognitive awareness significantly predicted achievement motivation, it did not significantly predict mathematics achievement. This finding should not be taken to mean that metacognition is unimportant in mathematics learning. Rather, it suggests that students' metacognitive awareness may not yet have developed into effective metacognitive regulation during actual problem solving. At the junior secondary level, students may know that planning, monitoring, and evaluating strategies are important, but they may not consistently apply these processes when working on mathematics tasks (Jeannotte & Kieran, 2017; Scheibling-Sève et al., 2020; Vamvakoussi, 2017). This distinction between awareness and enactment is critical. Prior studies have often found positive associations between metacognition and achievement, but such effects tend to be stronger when metacognition is examined in task-specific or performance-based contexts rather than through self-report measures alone. Therefore, the non-significant direct effect found here may reflect a gap between what students know about learning strategies and how effectively they use them in mathematics.

Achievement motivation also did not significantly predict mathematics achievement. This result suggests that stronger motivation did not automatically lead to better mathematics performance in this sample. One possible explanation is that students' motivation may have been oriented more toward external outcomes, such as scores or approval, than toward mastery of mathematical concepts. If so, motivation may have encouraged participation or effort, but not necessarily the kind of deep cognitive engagement needed for success in mathematics. Another explanation lies in the instructional context (Milenković & Stevanić, 2025; Rincón et

al., 2025). When classroom learning is strongly outcome-oriented, motivated students may still lack sufficient opportunities to develop reasoning, conceptual understanding, and strategic flexibility. This finding indicates that motivation alone is not enough. It must be supported by learning environments that channel students' effort into meaningful mathematical thinking.

The indirect effects also need to be interpreted carefully. Although the indirect coefficient of self-efficacy through achievement motivation was numerically larger than its direct coefficient, the pathway from achievement motivation to mathematics achievement was not significant. For this reason, the results do not provide strong evidence for a confirmed mediating role of achievement motivation. The same caution applies to metacognitive awareness. Its indirect effect through achievement motivation was relatively small, and the statistical basis for mediation remains weak (Khozaei et al., 2022; Rogowska et al., 2022). Therefore, the indirect effects identified in this study are better understood as indicative patterns rather than conclusive mediation. This is an important point because it prevents overinterpretation of the path model and keeps the discussion aligned with the empirical evidence.

Another important result is the contrast between the explanatory power of the two path models. The first model explained a substantial proportion of the variance in achievement motivation, whereas the second model explained only a small proportion of the variance in mathematics achievement. This suggests that self-efficacy and metacognitive awareness were useful in explaining students' motivational tendencies, but much less effective in explaining their actual mathematics performance. In other words, mathematics achievement in this study appears to have been shaped by a broader set of influences beyond the variables included in the model. Factors such as prior mathematical knowledge, instructional quality, mathematics anxiety, task difficulty, teacher support, and home learning conditions may have played more decisive roles (Hsieh et al., 2025; Kaiser et al., 2025; Olsher et al., 2025). This limited explanatory power should be acknowledged as an important boundary of the present findings.

These results carry clear implications for mathematics education. First, instructional practice should place greater emphasis on developing students' critical thinking because it was the only variable that showed a direct and significant relationship with mathematics achievement. Learning activities such as problem-based learning, open-ended tasks, reflective questioning, and argument-based discussion are therefore highly relevant. Second, self-efficacy and metacognitive awareness should still be strengthened, not because they directly improved mathematics achievement in this study, but because they contributed significantly to achievement motivation (Marufi et al., 2025). Third, efforts to enhance students' motivation should be accompanied by instructional designs that promote conceptual mastery and reasoning rather than merely encouraging effort or performance orientation. A balanced mathematics classroom should therefore integrate cognitive challenge, reflective learning, and psychological support.

This study also has limitations. The use of self-report questionnaires for self-efficacy, metacognitive awareness, and achievement motivation may not have fully captured how these constructs operate during actual mathematics learning. In addition, the study was conducted in one district and focused on Grade VIII students, which may limit the generalizability of the findings. The relatively low explanatory power of the second model also suggests that important variables were not included. Future studies should therefore consider incorporating additional predictors, such as prior knowledge, mathematics anxiety, instructional quality, and classroom climate, and may also benefit from using task-based or observational measures of metacognitive regulation. Despite these limitations, the present study contributes to the literature by showing that critical thinking remains the most relevant direct predictor of mathematics achievement, whereas self-efficacy and metacognitive awareness are more closely associated with students' motivational readiness than with achievement itself.

Conclusion

This study examined the direct and indirect relationships among critical thinking, self-efficacy, metacognitive awareness, achievement motivation, and mathematics achievement among eighth-grade students. The findings showed that critical thinking was the only variable that had a significant direct effect on mathematics achievement. In contrast, self-efficacy, metacognitive awareness, and achievement motivation did not significantly predict mathematics achievement. However, self-efficacy and metacognitive awareness significantly predicted achievement motivation, indicating that these variables were more closely associated with students' motivational readiness than with their actual mathematics performance. The indirect effects through achievement motivation were relatively weak, and the results did not provide strong support for a meaningful mediating role of achievement motivation. These findings suggest that improving mathematics achievement requires greater emphasis on strengthening students' critical thinking through learning experiences that promote reasoning, analysis, and reflective problem solving. Future research should include additional variables, such as prior knowledge, mathematics anxiety, instructional quality, and classroom climate, to provide a more comprehensive explanation of students' mathematics achievement.

Conflict of Interest

The author declares no conflict of interest.

Authors' Contributions

The first author, A.S., conceived the research idea presented and collected the data. S.A., M., and T. were the supervisors in this research who actively participated in the development of theory, methodology, organization and analysis of data, discussion of results, and approval of the final version of the work. All authors declare that the final version of this paper has been read and approved. The total percentage of contributions to the conceptualization, preparation, and correction of this paper are as follows: A.S.: 40%, S.A.: 15%, M.: 15% and T.: 30%.

Data Availability Statement

The author declares that the data supporting the results of this study will be made available by the corresponding author, [T.], upon reasonable request.

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