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Using Digital Pythagorean Tangram to Enhance Mathematical Justification Skills of Prospective Mathematics Teachers

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ABSTRACT

Mathematical justification is a crucial skill for mathematics teachers because it emphasizes proof and reasoning. This study examines how using the media "Digital Pythagorean Tangrams," a manipulable, multimedia tool on Polypad, can improve prospective teachers' justification skills when proving the Pythagorean theorem. This research used a one-group pretest-posttest design where students were given a pre-test, then received an intervention using the media, and subsequently completed a post-test. The participants included 44 mathematics education students at a university in Bandung. Instruments comprised a mathematical justification test (pretest and posttest) and Digital Pythagorean Tangram media. Pretest and posttest data were analysed to obtain the N-gain score to estimate the effect size. The N-gain data were analysed using a one-sample t-test to check whether the improvement was statistically significant. The results revealed an average N-gain score of 0.42, which falls into the medium category, demonstrating a notable improvement in students' mathematical justification skills before and after using the digital tangram media. The One-Sample t-test also indicated a significance level below 0.05, confirming that the increase in students' mathematical justification ability is significant. Integrating digital manipulatives into proof-oriented instruction can meaningfully enhance prospective teachers' justification skills and may serve as a practical pedagogical approach in mathematics teacher education.

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Introduction

Mathematical justification is a crucial skill for future teachers. Mathematical justification skill is a cognitive ability that involves providing logical reasoning and evidence to support mathematical propositions, solutions, or claims (Mata-Pereira & da Ponte, 2018; Staples et al., 2012). This skill helps teachers comprehend and convey complex mathematical concepts effectively (Hamidah, et al., 2025; Zeybek Simsek, 2025). It also enhances their pedagogical skills (Weingarden & Buchbinder, 2023), enabling them to facilitate students' learning (Mata-Pereira & da Ponte, 2018), boost their professional growth (Hamidah, Susiswo, Susanto, & Osman, 2025), and address educational disparities (Lai et al., 2023). The varied significance of justification underscores the importance of teacher education programs incorporating justification skills to produce high-quality math teachers.

Numerous studies find that prospective math educators struggle with the skills of mathematical justification. Future math teachers often struggle with abstract thinking, and their basic math knowledge is also not very solid (Isran et al., 2025) Both of them are essential for proof and argument construction. They might have advanced skills in conceptual knowledge, but when it comes to justification and relating the solution to a real-world example, they find it challenging (Dündar & Gündüz2017). A large number of future teachers can create logical proofs. However, when it comes to more complex methods like contradiction and contraposition, they struggle, which again points to a very shallow engagement with proof construction (Mukuka & Tatira, 2025). The solution to these problems requires an interactive teaching method, curriculum modifications, technology, and focused training programs (Anwar et al., 2025; Francisco, 2025). By these measures, teacher education programs will become better able to equip future math teachers to develop reasoning and argumentation skills in their own classes.

One effective way to help prospective students develop understanding and improve justification skills is to use manipulatives. Manipulative media allow learners to interact with mathematical objects, explore relationships among them, and develop critical and reasoning skills (Hakim et al., 2019; Utaminingsih et al., 2024). Tangram, a well-known manipulable media, has been widely used to help students grasp concepts of shape, area, and figure relationships through hands-on, exploratory activities (Cahyanita et al., 2021; Mufti et al., 2020). Studies have shown that Tangram can improve spatial abilities (Diaz Renavitasari & Afif Supianto, 2018; Eloy et al., 2020). The modified Tangram, into digital devices such as Scratch and mobile games, allows Tangram to be used by modern learners (Diaz Renavitasari & Afif Supianto, 2018). Therefore, Tangrams remain a significant educational tool, linking traditional teaching methods with modern technological advancements to make students' learning experience more engaging and effective.

Existing tangram research has mainly focused on understanding concepts and motivation to learn. No study has tested the digital tangram to improve students' mathematical justification skills. Additionally, previous studies mainly focus on students, with little attention to teachers or prospective teachers (Cahyanita et al., 2021; Mega Faniya et al., 2023; Wardani & Putra, 2024). Therefore, there is a gap in the literature on how tangram impacts justification skills, especially among prospective teachers. Theoretically, this research contributes to expanding the study of the relationship between digital media use, geometry, and mathematical justification skills, especially the ability of prospective teachers to justify and provide valid reasons. Practically, the results can guide lecturers and media developers in designing more interactive, reflective, and student-centered learning experiences that emphasize mathematical thinking processes. Based on these reasons, this research aims to explore the effect of Digital Pythagorean Tangram on prospective teachers' mathematical justification skills.

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Method

Type of Research

This research employs a quantitative approach, specifically a quasi-experimental design with a one-group pretest-posttest. This method was chosen because it is suitable for quantitative pre-experimental studies, such as testing the effectiveness of a specific program without a control group. The approach aims to determine the impact of using digital media Pythagorean tangram on the mathematical justification skills of prospective mathematics teachers. In terms of design, the one-group pretest-posttest study involves a single group and two assessments: a pretest and a posttest. First, the group takes a pre-test on mathematical justification questions, then receives an intervention using a digital media Pythagorean tangram. Finally, a post-test on the same topic is administered. The results from both assessments are then statistically analyzed through n-gain measurement and significance tests. In the intervention treatment section, the researcher used Digital Pythagorean Tangram media as shown in Figure 1. This media supports two display modes: teacher view and student view. From the teacher's perspective, all student activities during the tangram assembly are recorded and can be monitored in real time. Meanwhile, the student's view provides an interactive experience in using the tangram for learning. The concept of the tangram is based on proofs from Pythagoras' theorem, Bhaskara's proof, and Euclid's proof.

Figure 1. Front View of the Pythagoras Digital Tangram Puzzle

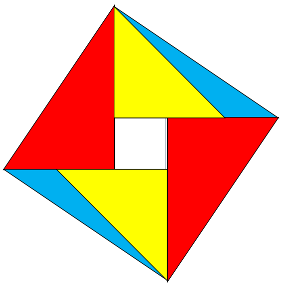
Population and Sample

The population in this study consists of prospective mathematics teachers, with a sample of 44 mathematics education students at a university in Bandung. Demographically, the participants are first-year students aged 18-19, with a gender ratio of 9 males to 35 females. Subjects were selected using a purposive sampling technique, based on students who had not yet taught courses related to the Pythagorean Theorem. The use of purposive sampling was intended to ensure that participants' understanding was based solely on their prior knowledge from secondary school.

Instruments

The instrument used is a test of mathematical justification ability. During development, this instrument was validated by an expert in algebra and geometry education. The expert validation scored 96 out of 100, indicating suitability for use. This instrument employs two indicators: decision-making (justification) and validation. The first indicator requires the subject to provide a correct or incorrect decision, a justification for the information, and an explanation of their reasoning. The second indicator asks the subject to give proof and reasons supporting the truth of a statement (Flegas & Charalampos, 2013). The complete problem-solving ability instrument is shown in Table 1

Table 1. Mathematical Justification Test

Tasks	Task Characteristics
Task #1. *Pretest* Given the following measurements of triangles, which ones are right-angled? Provide the reasons! 1. $\Delta ABC\ with\ AB = 105\ cm, BC = 360\ cm,\ and\ AC = 375\ cm$ 2. $\Delta PQR\ with\ PQ = 60\ cm, QR = 899\ cm,\ and\ PR = 901\ cm$ **Task #1.** *Posttest* Given the following measurements of triangles, which ones are right-angled? Provide the reasons! 1. $\Delta PQR\ with\ PQ = 50\ cm, QR = 624\ cm,\ and\ PR = 625\ cm$ 2. $\Delta ABC\ with\ AB = 120\ cm, BC = 225\ cm, and\ AC = 255\ cm$	Students are expected to make decisions by assessing whether a statement is true or false based on a Pythagorean triple or the general concept of the Pythagorean theorem.
Task #2. Look at the following picture. The square is composed of 6 triangles and one square. The red triangle is right-angled, and the yellow triangle is an isosceles right-angled triangle. If the red triangle has sides of a cm, b cm, and c cm, where c is the longest side. Prove that 1. The area of the red triangle = the area of the blue triangle + the area of the yellow triangle 2. Prove that $a^2 + b^2 = c^2$! 	Students should provide validation by presenting supporting evidence for the statement. This proof demonstrates the Pythagorean theorem as stated by Bhaskara.
Task #3. Based on the picture, prove the correctness of each of the following statements: - Area DBQE = area SRBC - Area ADEP = Area TCAU - Area ABQP = area SRBC + area TCAU	Students should provide validation by presenting supporting evidence for the statement. This proof demonstrates Euclid's Pythagorean theorem.

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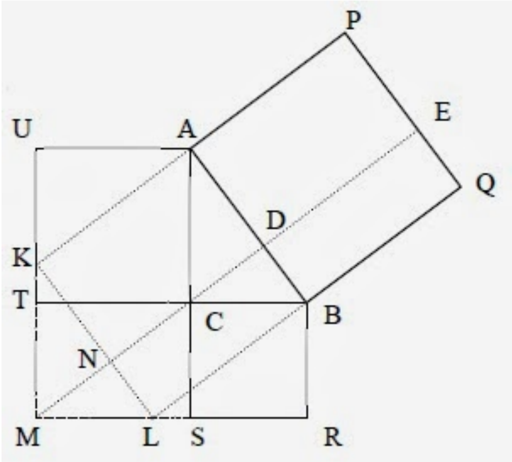

Data Collection

Data was collected through pretests and posttests in the form of tests of mathematical justification skills. The mathematical justification ability was used to determine the prospective mathematics teachers' ability to prove the Pythagorean theorem. This research was conducted at the beginning of the odd semester of the 2025/2026 academic year, namely in September 2025. The study utilized computer lab facilities to support the use of the Digital Pythagorean Tangram media. To enhance transparency and methodological rigor, the scoring rubric used in this study is presented in Table 2.

Table 2. Scoring Rubric for Mathematical Justification

Indicator	Description	Score	Performance Criteria
Decision Making (justification)	Students determine whether a statement is true or false using Pythagorean triples or the general form of the Pythagorean theorem.	3	Correct decision with appropriate application of the theorem or triple.
		2	Correct decision, but the justification is incomplete or unclear.
		1	Incorrect decision, but with some relevant reasoning.
		0	Incorrect decision with no relevant reasoning
Validation (Bhaskara's Proof)	Students justify the statement using evidence consistent with Bhaskara's area-based proof.	3	Provides a mathematically valid argument reflecting Bhaskara's rearrangement proof.
		2	Provides partially correct or incomplete evidence.
		1	Evidence is minimally related or contains significant errors.
		0	No relevant evidence provided.
Validation (Euclid's Proof)	Students justify the statement using evidence consistent with Euclid's geometric proof.	3	Presents a correct geometric argument consistent with Euclid's proof.
		2	Provides geometric evidence that is partially correct or incomplete.
		1	The evidence is incorrect or weakly connected to Euclid's proof.
		0	No relevant evidence provided.

Data Analysis

Data analysis was carried out using two types of statistics: descriptive and inferential. In the descriptive section, pre-test and post-test data were analysed to determine the mean, maximum, minimum, and standard deviation. The normalised gain (N-gain) was then calculated to assess improvement, categorised as low ($g < 0.3$), medium ($0.3 \leq g < 0.7$), and high ($g \geq 0.7$). Before performing inferential analysis, assumptions of normality and homogeneity were tested. For parametric tests, a One-Sample t-test was used to examine whether the average n-gain significantly differed from zero. Additionally, this research sought to estimate the effect size using Hedges' g to determine the extent of this media's use. All analyses were conducted using IBM SPSS Statistics Version 24. Potential threats to internal validity associated with the one-group design include maturation, testing effects, and instrumentation. These were acknowledged and addressed through standardised procedures and consistent use of digital media.

Research Findings

In this study, students' mathematical justification abilities were tested twice: through a pretest and a posttest. The average pretest score was 38.4, while the average posttest score was 63.63. To analyze whether there was a significant improvement in prospective mathematics teachers' mathematical justification abilities after learning with the Digital Pythagorean Tangram, the researchers used n-gain data from the students' mathematical justification abilities test. Through processing the n-gain data from the justification ability test of prospective mathematics teachers, several pieces of information were obtained: the n-gain data from 44 students had an average score of 0.42, a variance value of 0.086, meaning the measure of the spread of the data from the average was 0.086, a standard deviation value of 0.293, meaning the measure of how far the data deviates from the mean, was 0.293. The minimum n-gain value was -0.5, and the maximum n-gain value was 1. The complete descriptive statistical data are shown in Table 3.

Table 3. Descriptive Statistics (n=44)

	Pretest	Postest	N-Gain
Mean	38,40909	63,636364	,42053
Median	35	65	,39583
Variance	176,4799	384,14376	.086
Std. Deviation	13,28457	19,599586	,293038
Minimum	60	100	-,500
Maximum	15	30	1,000

Based on Table 3, the n-gain value of 0.42 is categorized as "medium" improvement ($0.3 \leq g < 0.7$), which means that the Digital Pythagorean Tangram was a significant, though not the best, factor in the enhancement of students' justification skills. Negative n-gain values observed in a few students (minimum = −0.50) indicate that some students did not perform as well on the posttest as on the pretest, which may be due to difficulties with interacting with digital media or stubborn misconceptions throughout the learning process.

As previously explained, this study examined whether the Digital Pythagorean Tangram improved prospective mathematics teachers' mathematical justification skills. To answer this question, a normality test was first conducted on the students' n-gain data using IBM SPSS Statistics Version 24. The hypotheses used in the n-gain data normality test are as follows:

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H_0: The n-gain data for the mathematical justification abilities of prospective mathematics teachers come from a normally distributed population.
H_1: The n-gain data for the mathematical justification abilities of prospective mathematics teachers come from a non-normally distributed population.

The testing criteria used in the n-gain data normality test are as follows:

If the probability (Sig.) is $\geq 0{,}05$, then H_0 is accepted.
If the probability (Sig.) is < 0.05, then H_0 is rejected.

Table 4 shows the output of the n-gain data normality test.

Table 4. Output of the N-Gain Data Normality Test

Tests of Normality						
	Kolmogorov-Smirnov[a]			Shapiro-Wilk		
	Statistic	df	Sig.	Statistic	df	Sig.
NGain	.094	44	.200*	.964	44	.190

*. This is a lower bound of the true significance.
a. Lilliefors Significance Correction

Because the sample size was less than 50, the researcher used the Shapiro-Wilk test to test the normality of the n-gain data. Using a significance level of $\alpha = 0.05$, The Shapiro-Wilk test yielded a probability value (Sig.) > 0.05,, indicating that H_0 is accepted. Therefore, the conclusion from the normality test is that the n-gain data from the mathematical justification ability test of prospective mathematics teachers are typically distributed. Thus, a parametric t-test, namely the One-Sample t-test, will be used to test the difference in the average n-gain against zero. Based on the One-Sample t-test of n-gain, the data results from SPSS are displayed in Figure 2.

Hypothesis Test Summary

	Null Hypothesis	Test	Sig.	Decision
1	The median of NilaiNGain equals 0,000.	One-Sample Wilcoxon Signed Rank Test	.000	Reject the null hypothesis.

Asymptotic significances are displayed. The significance level is .05.

Figure 2. One-Sample t-Test Output for N-Gain Data

The hypotheses used in the One-Sample t-test are as follows:

H_0: There is no significant improvement in prospective mathematics teachers' abilities to justify their mathematical reasoning after learning with the Digital Pythagorean Tangram.
H_1: There is a significant improvement in prospective mathematics teachers' abilities to justify their mathematical reasoning after learning with the Digital Pythagorean Tangram.

Statistically, the above hypotheses can be formulated as follows:

H_0: $\mu \leq 0$
H_1: $\mu > 0$

The testing criteria used in the One-Sample t-Test are as follows:

If the two-tailed p-value is $\geq 0{,}05$, then H_0 is accepted
If the two-tailed p-value is $< 0{,}05$, Then H_0 is rejected.

Using a significance level of $\alpha = 0{,}05$, Based on the results of the test of the difference in the mean n-gain against zero in one sample using the One-Sample t-Test, a sig. < 0.05 was obtained, meaning H_0 was rejected. Therefore, the conclusion obtained from the One-Sample t-Test results is that there was a significant improvement in the mathematical justification abilities of prospective mathematics teacher students after receiving learning using the Digital Pythagorean Tangram. Along with the hypothesis test, an effect size analysis (Hedges' g) was conducted, providing additional evidence of the intervention's effect. The combination of significance and effect size reporting will make the findings more robust and easier to interpret. The effect size, calculated as Hedges' g, is 1.55, based on the pretest and posttest means and standard deviations, which is categorized as a large effect size. This indicates that the Digital Pythagorean Tangram has significantly improved students' mathematical justification skills beyond what can be inferred from testing for significance alone.

Discussion

Previous studies support these results. In studies that used the Pythagorean puzzle for learning, this media was reported to improve students' mathematical problem-solving abilities (Mutmainah & Nuha, 2023) and their understanding of mathematical concepts (Sasmita et al., 2019). In other studies, the use of one type of puzzle, namely the tangram, in learning influences students' mathematical creative thinking abilities (Ridwan et al., 2023; Wardani & Putra, 2024) and their understanding of flat shapes (Ramadhani et al., 2024). However, in the context of mathematical justification, the current findings can also be aligned with established justification and proof frameworks. When viewed through the lens of Harel and Sowder's proof schemes, the improvement observed here reflects a shift from empirical reasoning to more structured deductive justification (Harel & Sowder, 1998).

The use of the Digital Pythagorean Tangram is effective for improving mathematical skills, particularly justification skills. This learning medium helps students better understand the Pythagorean Theorem. Students are not only accustomed to justifying a mathematical theorem, but they also learn to construct written proofs of the Pythagorean Theorem using various constructions using digital tangrams. In this study, the proof of the Pythagorean Theorem using tangrams presented in the Digital Pythagorean Tangram for students to explore includes proofs using Unit Construction, Bhaskara Construction, Euclid Construction, Liu Hui Construction, J. Versiuys Construction, J. Adams Construction, and J.E. Bottcher Construction. The decision-making and validation processes of students are supported by these dynamic, visual constructions, which clearly show geometric relationships, reduce unwarranted cognitive load, and allow students to check whether a claim is true or false directly. This tool helps connect intuitive reasoning with formal justification.

The Digital Pythagorean Tangram has been structured in a learning material format that not only includes tangrams as a learning medium to help students prove the Pythagorean Theorem. The Digital Pythagorean Tangram also includes prerequisite materials to help students recall the prior material needed to learn the Pythagorean theorem. Furthermore, the tangram constructions are structured in stages, from the simplest to the most complex. Detailed explanations are provided for several constructions, enabling students to understand the proof of the Pythagorean Theorem by correctly assembling the puzzle and by presenting the proof

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using the Digital Pythagorean Tangram. This research is one of the earliest studies to relate digital Pythagorean tangram explorations to the development of mathematical justification skills among prospective mathematics teachers, thereby making a contribution not explicitly mentioned in earlier literature.

During the learning process, the use of the Digital Pythagorean Tangram demonstrated that the researchers fully facilitated students' curiosity, creativity, and mathematical justification skills. The questionnaire results demonstrated that learning with the Digital Pythagorean Tangram increased their enthusiasm for proving the Pythagorean Theorem through tangram construction and for studying written proofs of the Pythagorean Theorem. The variety of available constructions led them to realize the many ways to prove the Pythagorean Theorem, thereby encouraging them to study each construction individually. In fact, the researchers did not experience any significant obstacles during the learning activities, because the students were genuinely focused on following the learning, engrossed in arranging the tangrams in the correct order, actively understanding each flow of the Pythagorean Theorem proof, and actively asking questions when there were things that were difficult or not understood. It is also supported by results from other studies that use tangrams in mathematics learning, where students feel happy, enthusiastic, and motivated to participate because it not only involves learning but also includes elements of play that remain meaningful (Mega Faniya et al., 2023). Still, the conclusions to be drawn from these results should be made with caution, as the one-group pretest–posttest design used in this research has inherent drawbacks, including potential testing effects and the lack of a control group.

Conclusion

Based on the results and discussion, the Digital Pythagorean Tangram has been shown to improve prospective mathematics teachers' mathematical justification skills significantly. As such, the evidence lies in the increase of the average score from 38.4 in the pretest to 63.63 in the posttest, with a moderate n-gain of 0.42 and a statistically significant One-Sample t-test result ($p < 0.05$) being responsible for this change. The increased ability to provide mathematical justification, as demonstrated by the n-gain scores, results from their manipulation of media and efforts to find reasons for it. The influence has also been shown to have a significant effect, as evidenced by the Hedges' g score of 1.55. These findings can serve as a theoretical foundation for developing interactive and digital media for students. This is because the use of digitally created tangram designs also boosts the potential to turn proof into not just a thinking process but also an engaging game that attracts students. Teachers can also use this media as an activity that supports reasoning and creativity.

The researcher admits that this study still has limitations, including the use of a one-group research design. The study also focused on a single cognitive domain, namely the proof of the Pythagorean theorem, and involved students from only one university, restricting generalizability. A task-level analysis of how students used different tangram constructions was not conducted, which limits insight into process-level learning gains. Furthermore, the article notes that no task-level analysis was conducted on how students used different tangram constructions. Researchers in the future should consider using different or experimental designs, including multiple raters to ensure scoring reliability, and diverse mathematical tasks and participant groups. On the practical side, the discoveries show that instrumental geometric digital tools, such as the Digital Pythagorean Tangram, can be used in teacher education to deepen reasoning instruction, facilitate the exploration of multiple proof structures, and motivate students to engage in mathematical justification.

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Conflict of Interest

The authors declare that there is no conflict of interest.

Authors' Contributions

All authors conceived the research idea and collected the data. Y.A. contributed to the conceptualization of the article, theory development, and methodology. Two other authors, N.S.P. and B.C.R.B., actively contributed to data organization and analysis, and discussed the results. All authors have read and approved the final version. Contribution percentages are: Y.A.: 40%, N.S.P:. 30%, and B.C.R.B.: 30%.

Data Availability Statement

The authors declare that data sharing is not applicable, as no new data were created or analyzed in this study.

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