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Analysis of Students' Thinking Processes in Solving Mathematical Problems Based on Bruner's Theory in Pythagorean Theorem Tasks

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ABSTRACT

This study aims to describe and analyze the thinking processes of junior high school students in solving mathematical problems based on Bruner's theory in the context of the Pythagorean theorem. The study was conducted at SMP Negeri 7 Kota Jambi in May 2025. Data were collected through classroom observations as preliminary data, followed by questionnaires and problem-solving tests as primary data, and interviews as supporting data. All data were analyzed qualitatively. The findings show that at the enactive stage, most students demonstrated low performance; four students were categorized as poor, one as excellent, and one as good. At the iconic stage, student performance ranged from low to adequate, with one student rated excellent, two rated adequate, and three rated poor. At the symbolic stage, most students demonstrated high performance, with three students rated excellent, two rated good, and one rated poor. These results indicate that although students generally succeed at the symbolic stage, weaknesses at the enactive and iconic stages suggest that their understanding remains predominantly procedural rather than conceptual. Teachers are encouraged to apply learning strategies that intentionally strengthen all three of Bruner's representational stages, rather than moving directly to symbolic reasoning without first providing concrete experiences and visual representations. Teachers should also consider students' diverse thinking processes when designing instruction. Future studies should include larger samples and more diverse methods, such as quantitative or mixed-methods approaches, to further examine how representation-based strategies influence students' mathematical problem-solving performance.



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Introduction

In essence, education is a deliberate and organized effort to create an environment and learning process in which students actively develop their capacity for religious spiritual strength, self-control, intelligence, noble character, and the skills needed by themselves, society, the nation, and the state (Qudsiyah et al., 2023). Education is not merely a process of conveying information and training skills, but also a systematic effort to channel desires, fulfill needs, and explore the potential of everyone so that they can live satisfying personal and social lives. Education should not be positioned only as preparation for the future, but also as part of children's current life experiences during their developmental process (Qudsiyah et al., 2023). Education is a conscious and well-planned effort to create a learning environment and learning process that enables students to actively develop their potential, so that they have religious spiritual strength, self-control, strong character, intelligence, noble morals, and the skills needed for themselves and their surrounding communities (Rahman et al., 2022).

The objectives of education outlined in Law No. 20 of 2003 concerning the National Education System emphasize that education serves to explore and develop students' potential so that they grow into individuals who believe in and fear God Almighty, have noble character, are physically and spiritually healthy, knowledgeable, skilled, creative, independent, and become democratic and responsible citizens (Halid, 2024). Thus, it can be concluded that there are two main aspects in education, namely the development of human potential and the inheritance of culture across generations. Education is an activity that involves many individuals who demonstrate educated behavior (Amaliyah & Rahmat, 2021). Education must be implemented to revive the values of Pancasila and prepare graduates who possess a strong sense of nationality when facing the era of globalization (Hatip & Setiawan, 2021). Therefore, a mental revolution in education needs to be a continuous and ongoing process to create sustained quality improvement oriented toward shaping the future of humanity based on national cultural values (Mulyasa, 2017). Education has a significant influence on the progress and growth of a country. Likewise, the quality of education can be seen from the learning that is implemented. The implementation of appropriate learning can create meaningful outcomes, especially in mathematics (Purba, 2022).

Etymologically, mathematics refers to knowledge gained through reasoning. It is formed from human thought related to processes and reasoning. Mathematics plays an important role across various fields of knowledge. It is a fundamental discipline encountered at every level of education, from elementary school to university. Mathematics is a precise field of study that cannot be interpreted in multiple ways, which is one of its advantages compared to other subjects. However, it is still perceived as a difficult subject by many students, often resulting in poor academic performance. This issue can be overcome if students possess adequate problem-solving skills (Andriani, 2023). Students need problem-solving skills to address challenges not only in mathematics but also in other subjects and daily life (Astutiani & Hidayah, 2019). Individuals with strong problem-solving skills can think analytically, make better decisions in daily life, and improve their critical thinking abilities when facing various situations (Agustiani et al., 2022). Problem solving refers to the type of thinking required to achieve goals that are not immediately attainable and requires the use of higher-order thinking processes (Farid, 2024). Students must employ structured and logical thinking to choose the best strategies for achieving their goals. Problem solving is inseparable from the thinking process; thus, students with stronger thinking abilities tend to solve problems more effectively (Sukmawati & Rakhmawati, 2023).

Mathematics is closely related to the thinking process as a method for solving problems. Thinking is a mental activity that arises when an individual is faced with a problem, then

formulates, handles, and understands it (Saputra, 2024). Thinking is defined as a series of cognitive processes involving the manipulation of knowledge. This process may lead to conclusions or solutions to problems (Mailani, et al., 2022; Limbach & Waugh, 2020). Typically, thinking begins with acquiring knowledge, then processing, storing, and retrieving relevant information from memory when needed (Dewi et al., 2023). Through thinking, students learn to connect information to construct new knowledge, formulate hypotheses, test their validity, and draw conclusions based on the results.

Thinking can be described as a dialectical process, meaning that during thinking, a question-and-answer process occurs internally (Kholid et al., 2021; Khoiriyah & Murni, 2021). Through thinking, individuals develop the intelligence and ability to distinguish right from wrong or good from bad (Wibowo, 2024). The mind plays a crucial role in this process. As educators, we must develop students' thinking abilities alongside other learning aspects. Each student has different thinking abilities. A person is said to think when engaging in mental activities, and those who study mathematics inevitably engage in such activities. In learning mathematics, thinking processes always occur, leading to differences among students (Gaol et al., 2022).

According to Sansena (2022), Piaget stated that human cognitive development occurs in four successive stages: (1) the sensorimotor stage (0–2 years), (2) the preoperational stage (2–7 years), (3) the concrete operational stage (7–11 years), and (4) the formal operational stage (11 years and above). In the formal operational stage, individuals are able to think abstractly and logically and engage in hypothetical-deductive reasoning. This ability allows them to understand non-concrete concepts and design solutions for complex problems. Considering that the research subjects are eighth-grade junior high school students aged around 14–15 years, the formal operational stage is most relevant to this study.

One of the mathematics topics requiring strong thinking processes is the Pythagorean theorem, which relates to triangles and their side lengths. Solving problems involving this topic requires students to visualize shapes and perform precise calculations. Therefore, students must use logical and analytical thinking to connect relevant information and solve problems accurately. In this study, the researcher used Bruner's cognitive theory to observe the stages of students' thinking. This theory was chosen because it aligns with field conditions and reflects Bruner's principles, such as organization, notation, contrast and diversity, and connection. Bruner divides the thinking process into three stages: (1) the enactive stage, where students learn through direct experiences or real objects; (2) the iconic stage, where students use pictures, diagrams, or visual representations; and (3) the symbolic stage, where students use abstract symbols, including mathematical notation. These three stages form the basis for observing how students solve mathematical problems on the Pythagorean Theorem.

The researchers conducted a problem-solving test to identify students' thinking processes. The results showed that students in class VIII F demonstrated varied thinking abilities. In line with the researchers' objectives and based on observations at SMP Negeri 7 Kota Jambi, students in class VIII F indeed displayed diverse thinking processes. The following figures present examples of student responses used to assess the extent of their thinking processes based on Bruner's theory.



Figure 1. Problem-Solving Ability Test Results



Figure 2. Problem-Solving Ability Test Results

Based on these responses, the researcher assessed variations in students' thinking processes. In question 1, students were asked to write the formula for determining the length of a triangle's side. While their responses were not entirely correct, the students understood what to write based on the visual representation provided. According to Bruner's theory, their thinking process had reached the iconic stage but had not yet reached the symbolic stage because they were able to visualize the problem but struggled to determine the next steps. Meanwhile, in another student's response to question 3, where the task was to determine the length of one triangle side, the student solved the problem using mathematical symbols correctly. This indicates that the student was able to interpret the triangle image, connect it to the Pythagorean theorem, and solve the problem. According to Bruner's theory, this student had reached the symbolic stage after previously demonstrating the iconic stage. These two responses show clear differences in students' thinking processes. Based on observations and interviews, it was found that most eighth-grade students still experience difficulties in solving mathematical problems, particularly those involving the Pythagorean theorem and its application to two-dimensional shapes. Students often feel confused about determining the correct steps and appropriate formulas. Therefore, the researcher aimed to examine students' thinking processes when solving problems involving this topic.

Method

Type of Research

The type of research used in this study is qualitative research with a descriptive approach, which describes the analysis of students' thinking processes in solving mathematical problems based on Bruner's theory related to the Pythagorean Theorem. Sugiyono (2015) explains that qualitative research aims to understand phenomena experienced by research subjects by exploring data through in-depth interviews, observations, and documentation. This research is descriptive, with the intention of obtaining a deep and holistic understanding of an event or phenomenon. Based on this definition, the research conducted is descriptive qualitative research. This study describes students' thinking processes in solving mathematical problems based on Bruner's theory, enabling the researcher to determine whether their thinking has reached the enactive, iconic, or symbolic stage based on their solutions to Pythagorean Theorem problems. Sugiyono (2015) also states that qualitative research is an approach used to explore and understand human experiences in various social and cultural contexts. This research emphasizes the use of multiple forms of qualitative data, such as interviews, observations, and documents, all of which are analyzed to identify the central themes in the study. Qualitative research aims to understand phenomena experienced by research subjects holistically through verbal descriptions within a natural context and through the use of various scientific methods. The analytical component of this study focuses on identifying students' thinking processes in solving mathematical problems based on Bruner's theory on the Pythagorean Theorem in Grade VIII at SMP Negeri 7 Kota Jambi.

Participants

Data is a collection of facts in the form of numbers, letters, images, or sounds that do not yet have meaning or interpretation (Kurniawan et al., 2023). The data in this study were obtained from written tests, questionnaires, and interviews, which were then processed to identify the strategies or cognitive steps used by students and their thinking processes in solving mathematical problems based on Bruner's theory related to the Pythagorean Theorem. Therefore, the data collected in this study consist of students' answers from written tests, responses to questionnaires, and verbal statements obtained from interviews between the researcher and the research subjects. A widely used technique for selecting subjects in qualitative research is purposive sampling (Sugiyono, 2015). Purposive sampling is a technique for selecting data sources based on specific considerations, such as choosing individuals who are most knowledgeable or most relevant to the phenomenon being studied. Research subjects act as sources of information that will be explored to uncover facts in the field. In this study, the sample consisted of 30 students from class VIII F of SMP Negeri 7 Kota Jambi. Class VIII F was selected based on the recommendation of teachers at SMP Negeri 7 Kota Jambi and because students in this class showed diverse thinking processes in problem solving, as indicated by the results of a problem-solving ability test based on Bruner's theory. Therefore, eighth-grade students in class VIII F were chosen as research subjects to observe their thinking processes in solving mathematical problems based on Bruner's theory related to the Pythagorean theorem. The determination of research subjects in qualitative studies is not based on the number of individuals needed, but on who among them is most involved in the phenomenon and possesses the key information required. The prospective subjects in this study were six eighth-grade students from SMP Negeri 7 Kota Jambi who were categorized into the enactive, iconic, and symbolic stages based on written tests and questionnaire results aligned with Bruner's theory. They were then grouped into pairs to facilitate deeper investigation by the researcher.

Instruments

In this study, a written test was used to determine students' thinking processes in solving mathematical problems related to the Pythagorean theorem. The instrument used was a problem-solving ability test designed to identify students' cognitive steps when working through mathematical tasks. The problem-solving test consisted of three essay questions developed based on problem-solving indicators and Bruner's theory. The test instrument used in this study was an essay-based worksheet on Pythagorean theorem problems for eighth-grade junior high school students. Essay questions were chosen because they require students to provide detailed solutions, enabling the researcher to observe each step taken in the problemsolving process and to capture an overview of their thinking processes. According to Nur & Utami (2022), an interview is a technique for obtaining information through direct communication between the interviewer and the informant, aimed at exploring personal understanding or experiences related to events or issues relevant to the research. Various types of interviews may be used in qualitative studies. In this research, a semi-structured interview was employed. The interviews in this study were conducted to verify students' responses on the problem-solving test and to confirm the results of the questionnaire regarding their thinking processes at each stage of Bruner's theory. In this study, questionnaires on thinking processes and problem solving were distributed to determine students' problem-solving abilities and thinking processes based on Bruner's theory. The questionnaire results were categorized by examining the stages in Bruner's theory where students obtained higher scores. For example, higher scores on statements related to the use of concrete objects or imagining direct physical

activities indicate that the student is categorized at the enactive stage. If high scores appear in more than one stage, the student may be categorized into a combination of Bruner's representational stages.

Procedures

The importance of research procedures lies in the application of appropriate methods to obtain accurate data. These procedures include not only standard steps such as problem identification, research design, data collection, data analysis, and drawing conclusions, but also the testing of the validity and reliability of research instruments. Research procedures refer to the stages carried out throughout the research process. In this study, the research procedures consisted of three stages, namely the pre-field stage, the fieldwork stage, and the data analysis stage. Activities conducted at the pre-field stage began with consultation with the supervising lecturer regarding the proposed title. Permission was then obtained to conduct observations at the selected school, SMP Negeri 7 Kota Jambi, followed by an initial observation of the students' conditions. A problem-solving ability test was administered to determine differences in students' thinking processes when solving mathematical problems. The research instruments were prepared, including a problem-solving ability test, a questionnaire based on Bruner's theory, and an interview guide to identify students' stages of thinking in solving mathematical problems. The research proposal was then submitted, and the selection of the research school, SMP Negeri 7 Kota Jambi, was finalized. Permission to conduct the study was obtained, and an agreement was reached with the principal and mathematics teachers regarding the classes to be involved, the research schedule, and the materials to be used. Finally, the official research permit was submitted.

In the fieldwork stage, the first step was to select the research subjects by administering a questionnaire to identify their thinking processes in problem solving at the enactive, iconic, and symbolic stages in accordance with the predetermined criteria and research objectives. A problem-solving test consisting of Pythagorean theorem questions was then given to analyze students' thinking processes based on Bruner's theory. This was followed by interviews to explore and verify the students' written responses. The written answers and verbal responses obtained from the interviews were assessed for consistency. Afterward, the answer sheets were evaluated based on established scoring guidelines, and all collected data underwent analysis, including data reduction, data presentation, and conclusion drawing. Finally, data validity was tested using source triangulation and technique triangulation.

Analysis

Data analysis is the process of examining collected data using appropriate statistical or qualitative techniques to identify patterns, relationships, or trends to generate meaningful information for research (Rosyidah & Masykuroh, 2024). According to Waruwu (2024), data analysis is a research step aimed at processing and interpreting the data that has been collected to identify patterns or draw conclusions that can answer the research questions. Data analysis in this study was carried out using a qualitative approach to secondary data in the form of case studies and previous research used to support the focus of the study. Researchers collected and analyzed data simultaneously, using methods that allow research questions to be refined and adjusted throughout the ongoing data collection and analysis process. This iterative process continued until data saturation was reached, indicating that no new relevant information had emerged. Thereafter, data analysis involved data reduction, data display, and conclusion drawing or verification.

Results

Description of survey results data

To analyze students' thinking processes in solving mathematical problems based on Bruner's theory on the Pythagorean theorem, the first step taken by the researcher was to give a questionnaire on thinking processes in problem solving based on Bruner's theory to eighth-grade students at SMP Negeri 7 Kota Jambi. This research questionnaire was developed based on the stages in Bruner's theory, namely the enactive, iconic, and symbolic stages. The research questionnaire was assessed using a rating scale, namely VS (Very Agree), A (Agree), DA (Disagree), and DS (Strongly Disagree). The questionnaire was administered to obtain research subjects, namely students from that class who were students with thinking processes that were broadly at the enactive, iconic, and symbolic stages. The questionnaire was administered on May 19, 2025, to 30 students in class VIII F.

This questionnaire contains statements that can describe students' thinking processes when faced with a mathematical problem. The questionnaire used in this study is based on Bruner's theory, which is adapted to the circumstances of students when faced with mathematical problems on the Pythagorean theorem material, and has been validated and approved by a mathematics education lecturer who has validated the questionnaire and declared it valid. This questionnaire consisted of 14 questions, which were then divided based on the stages in Bruner's theory and a mixed stage, which was a combination of the three stages in Bruner's theory. Statements 1 to 3 described the enactive stage, statements 4 to 7 described the iconic stage, statements 8 to 11 described the symbolic stage, and statements 12 to 14 described the mixed stage. The questionnaire was filled out by checking the boxes provided for each statement. The results of this questionnaire were then confirmed by the researcher with the mathematics teacher who taught class VIII F. From the results obtained from filling out the questionnaire, in class VIII F, out of 30 students in class VIII F, there were 2 students in the enactive stage, 2 students in the iconic stage, 2 students in the symbolic stage, 12 students in the iconic and symbolic stages, and 12 students in the mixed stage.

In this study, the researcher only studied students who were in the enactive, iconic, and symbolic stages. Thus, the number of subjects to be studied in this study was 6 people. The selection of the six research subjects was based on the results of the questionnaires filled out by the students and also using purposive sampling techniques. Purposive sampling is a sampling technique with specific considerations and objectives. In this study, purposive sampling was used by considering students who were at the enactive, iconic, and symbolic stages. In addition, the sample was also determined based on the recommendations of the eighth-grade mathematics teacher, who understood the characteristics of the students better, thus making it easier for the researcher to select subjects that were in line with the research objectives. By using purposive sampling, it was hoped that a sample that represented the research objectives and met the criteria for providing information would be obtained.

Table 1. Research Subjects based on Questionnaires.

No	Stage	Student Initials
1	Enactive	FR
		NZ
2	Iconic	MF
		AN
3	Symbolic	RA

MR

Based on the questionnaire results in Table 1, it can be seen that each stage of Bruner's theory is represented by two students. This shows that all stages of enactive, iconic, and symbolic thinking are represented in the study, so that the analysis can describe the variations in students' thinking processes comprehensively.

Problem-Solving Ability Test Results based on Bruner's Enactive Stage Theory on Question Number 1 Subject FR

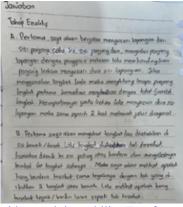


Figure 3. Results of the Problem-Solving Ability Test for Subject FR Number 1

In the problem identification indicator, the FR subject reads the question carefully to understand its meaning, which is reinforced through the interview results. In the solution planning stage, FR is able to design problem-solving steps and answer questions according to the enactive stage.

- *P:* What is the first step you take to solve the problem?
- FR: First, I read all the questions given so that I can understand what is meant in the question.
- P: After reading all the questions, which question do you think is easier to understand and work on?
- NZ: Of all the questions, I think question no. 1 is easy to understand and work on.
- P: Why question no. 1?
- NZ: Because I find it difficult to understand the other questions, sir, whereas I can understand question no. I and can solve it by explaining it.

In solving the problem, FR successfully answered the question correctly using enactive thinking, which is imagining the activity directly without the help of pictures or the Pythagorean theorem. In the evaluation stage, FR reviewed his answers by reading and matching the results with the problems, as confirmed through interviews.

Problem-Solving Ability Test Results based on Bruner's Iconic Stage Theory on Question Number 2 Subject FR

In the problem identification indicator, FR subjects read questions carefully to understand their meaning, as confirmed by interview results. In the solution planning stage, FR is able to devise problem-solving steps and answer questions according to the iconic stage.

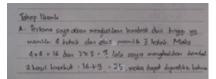


Figure 4. Results of the Problem-Solving Ability Test for Subject FR Number 2

P: After reading question number 2, what did you do?

FR: I tried to understand it and think about the answer to fill in, like number 2, which is almost similar to number 1 without using a formula.

However, in the problem-solving stage, FR's answers to questions 2a and 2b were not fully in line with the iconic stage because they still used calculations and the final results were not entirely accurate. In the evaluation stage, FR reviewed their answers by reading and matching them with the problems given, as confirmed by the interview results.

Problem-Solving Ability Test Results based on Bruner's Symbolic Stage Theory on Question Number 3 Subject FR

In the problem identification indicator, FR subjects read questions carefully to understand their meaning, as confirmed by interview results. However, in the solution planning stage, FR was unable to devise problem-solving steps as intended in the questions.

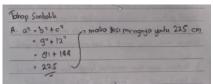


Figure 5. Results of the Problem-Solving Ability Test for Subject FR Number 3

P: After reading question number 2, what did you do?

FR: I tried to understand and think about the answer, sir, but I didn't really know the formulas that would be used.

In solving the problem, FR only answered question 3a, but the result was not perfect and there were still errors in the final answer. However, during the evaluation stage, FR reviewed his answers by reading and matching them with the problems, as confirmed through an interview.

Problem-Solving Ability Test Results based on Bruner's Enactive Stage Theory on Question Number 1 Subject NZ

On the indicator of identifying problems, the NZ subject was rated as good because he read the questions carefully and identified problems in a way that was easy to understand, as confirmed by the interview results.

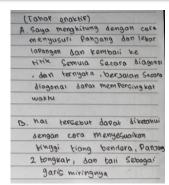


Figure 6. Results of the Problem-Solving Ability Test for Subject FR Number 1

P: What was the first step you took to solve question 1?

NZ: First, I read the question carefully and decided what steps to take to solve it. Not being allowed to use formulas made me a little confused, sir.

During the planning stage, NZ was able to devise problem-solving steps and answer questions according to the enactive stage. In solving the problem, NZ successfully completed questions 1a and 1b by imagining the activity directly without using pictures or the Pythagorean theorem. During the evaluation stage, NZ reviewed their answers by reading and matching them with the problem, as confirmed by the interview results.

Problem-Solving Ability Test Results based on Bruner's Iconic Stage Theory on Question Number 2 Subject NZ

On the indicator of identifying problems, NZ subjects read questions carefully and were able to recognize problems so that they were easy to understand, as confirmed by the interview results.

	(Tahap (konik)
A	y2 = 82 + 42
	= 9 + 16
	= 1/25
	= 5,
	"
8	42 = 62 +82
	= 36 +64
	= 100
	y = 104

Figure 7. Results of the Problem-Solving Ability Test for Subject FR Number 2

P: What was the first step you took to solve question 2? NZ: First, I read the question carefully and decided what steps to take to solve it.

In the planning stage, NZ applied this well and answered the question according to the iconic stage. However, in the problem-solving stage, NZ's answers to questions 2a and 2b were not in accordance with the iconic stage because the solution still used a formula. In the evaluation stage, NZ still checked their answers by reading and matching them with the problems, as confirmed by the interview results.

Problem-Solving Ability Test Results based on Bruner's Symbolic Stage Theory on Question Number 3 Subject NZ

On the problem identification indicator, NZ read the questions carefully and was able to identify the problems so that they were easy to understand, as confirmed by the interview results. At the solution planning stage, NZ was able to devise problem-solving steps and answer questions according to the symbolic stage.

	Nabil 2harif
	VIII 9
	(Tahap Gimbolik)
A	y2 = 92 + 122
	y2 = 81 + 144
	= √225
	· y = 15,
	"
8	26 ² = 7 ² + 6 ²
	$b^2 = 2b^2 - 7^2$
	b2 = 625 - 49
	b = 1576
	b = 24,
	4

Figure 8. Results of the Problem-Solving Ability Test for Subject FR Number 3

P: What was the first step you took to solve the problem?

NZ: First, I read the question carefully and determined the formula to be used to solve it.

In solving the problem, NZ successfully completed questions 3a and 3b correctly using symbols or formulas without the aid of pictures, in accordance with the symbolic stage. In the evaluation stage, NZ reviewed his answers by reading and matching them with the problems, as confirmed through interviews.

Problem-Solving Ability Test Results based on Bruner's Enactive Stage Theory on Question Number 1 Subject MF

In the problem identification indicator, MF read the question but did not understand the meaning of the question, as confirmed by the interview results. In the solution planning stage, MF was unable to devise problem-solving steps according to the question.

P : What was the first step you took to solve the problem?

MF: I read question no. 2, but I didn't know what the first step was because I wasn't allowed to use formulas.

P: After completing the question, did you reread your answers before submitting them? MF: For question 1, no, sir, because I didn't answer that question.

In solving the problem, MF did not provide answers to questions 2a and 2b because they had difficulty imagining the activities directly, so they were assessed as insufficient. At the evaluation stage, MF also did not check or realize their mistakes because there were no calculation results, as confirmed by the interview.

Problem-Solving Ability Test Results based on Bruner's Enactive Stage Theory on Question Number 2 Subject MF

On the problem identification indicator, the subject MF read the questions carefully and was able to identify the problems to determine which questions were easy to understand, as confirmed by the interview results.

P : What was the first step you took to solve the problem?

MF: I read question no. 2 carefully and determined a plan to solve the problem based on what I knew from the question.

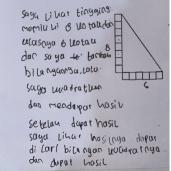


Figure 9. Results of the Problem-Solving Ability Test for Subject MF Number 2

At the planning stage, MF was able to devise problem-solving steps and answer questions in accordance with the iconic stage. In solving problems, MF worked on questions 2a and 2b, although not perfectly, but in accordance with the iconic stage because he used pictures without symbols or formulas. At the evaluation stage, MF was rated very good because he reviewed his answers by reading and matching them with the problems, as confirmed by the interview results.

Problem-Solving Ability Test Results based on Bruner's Enactive Stage Theory on Question Number 3 Subject MF

In the problem identification indicator, MF read the question but did not understand the meaning of the question, as confirmed by the interview results. In the solution planning stage, MF was unable to devise problem-solving steps as intended in the question.

P : What was the first step you took to solve the problem?

MF: I read question no. 3, but I didn't know what the first step was because there was no picture like in question no. 2.

P: After completing the question, did you reread your answers before submitting them? MF: For question 3, no, sir, because I didn't answer that question.

In solving the problem, MF did not provide answers to questions 3a and 3b because of difficulty imagining the activities directly, so it was considered insufficient. At the evaluation stage, MF also did not check the answers or realize the mistakes, as confirmed by the interview results.

Problem-Solving Ability Test Results based on Bruner's Enactive Stage Theory on Question Number 1 Subject AN

In the problem identification indicator, subject AN read the question but did not understand the meaning of the question, as confirmed by the interview results. In the solution

planning stage, AN was unable to devise problem-solving steps in accordance with what was intended in the question.

- *P* : What was the first step you took to solve the problem?
- AN: I read question no. 1, but I didn't know what steps to take to solve this problem, sir.
- *P* : After completing the question, did you reread your answers before submitting them?
- *AN*: For question 1, no, sir, because I didn't answer that question.

In solving the problem, AN did not provide answers to questions 1a and 1b because they had difficulty imagining the activities directly, so they were assessed as insufficient. At the evaluation stage, AN also did not check their answers or realize their mistakes, as confirmed by the interview results.

Problem-Solving Ability Test Results based on Bruner's Enactive Stage Theory on Question Number 2 Subject AN

In the problem identification indicator, AN read the questions but did not understand their meaning, as confirmed by the interview results. In the solution planning stage, AN was unable to devise problem-solving steps in accordance with the content of the questions.

- P: What was the first step you took to solve the problem?
- AN: I read the questions given, sir, but I was confused about question no. 2 because we were not allowed to use formulas.
- P: After completing the question, did you reread your answers before submitting them? AN: For question 2, no, sir, because I did not answer that question.

In solving the problem, AN did not provide answers to questions 2a and 2b because they had difficulty imagining the activities directly, so they were assessed as insufficient. At the evaluation stage, AN also did not check their answers or realize their mistakes, as confirmed by the interview results.

Problem-Solving Ability Test Results based on Bruner's Enactive Stage Theory on Question Number 3 Subject AN

In the problem identification indicator, subject AN read all questions carefully and was able to identify problems to determine which questions were easy to understand, as confirmed by the interview results. In the solution planning stage, AN was able to devise problem-solving steps and answer questions according to the symbolic stage.

- P: After reading all the questions, which question do you think is easier to understand and work on?
- AN: Of all the questions, I think question no. 3 is easier to understand and work on.
- P: Why question no. 3?
- AN: Because I find it difficult to understand the other questions, whereas I can understand question no. 3 and can work on it using the Pythagorean theorem.

```
a. Dik: b=g em

c:12 em

Dit: a:..?

Jub:

a^2 = b^2 + C^2

a^3 = 8^3 + C^2

a^4 = 81 + 199

a^2 = 225

a = 15 cm

Jadi Sisi Miringnya adalah 15 cm

b. Dik: a: 7 cm

c: 25 cm (c sebegai sisi miring)

Jub:

b^2 = C^2 - a^2

b^3 = 25^2 - 49

b = 576

b = 576

b = 576

b = 576

b = 576
```

Figure 10. Results of Subject AN Number 3's Problem-Solving Ability Test

In solving problems, AN correctly worked out questions 3a and 3b using symbols or formulas without the aid of pictures, in accordance with the symbolic stage. At the evaluation stage, AN reviewed their answers by reading them and matching them with the problems, in accordance with the confirmation of the interview results.

Problem-Solving Ability Test Results based on Bruner's Enactive Stage Theory on Question Number 1 Subject RA

In the problem identification indicator, RA read the question but did not understand the meaning of the question, as confirmed by the interview results. In the solution planning stage, RA was unable to devise problem-solving steps in accordance with the content of the question.

P: What was the first step you took to solve the problem?

RA: I read question no. 1, but I didn't know what to do because I wasn't allowed to use formulas.

P: After completing the question, did you reread your answers before submitting them?

RA : No, sir, because I didn't answer that question.

In solving the problem, RA did not provide answers to questions 1a and 1b because of difficulty imagining the activities directly, so it was considered insufficient. At the evaluation stage, RA also did not check the answers or realize the mistakes, as confirmed by the interview results.

Problem-Solving Ability Test Results based on Bruner's Enactive Stage Theory on Question Number 2 Subject RA

On the indicator identifying problems, the subject RA read the question but did not understand the meaning of the question, as confirmed by the interview results. At the stage of planning the solution, RA was not yet able to design problem-solving steps according to the content of the question.

P : What was the first step you took to solve the problem?

RA: I read question no. 2, but I didn't know what to do because I wasn't allowed to use formulas like in no. 1.

P: After completing the question, did you reread your answers before submitting them? RA: No, sir, because I didn't answer that question.

In solving the problem, RA did not provide answers to questions 2a and 2b because he had difficulty imagining the activities directly, so he was assessed as lacking. At the evaluation stage, RA also did not check his answers or realize his mistakes, as confirmed by the interview results.

Problem-Solving Ability Test Results based on Bruner's Enactive Stage Theory on Question Number 3 Subject RA

On the indicator of identifying problems, the subject RA read the questions carefully and was able to recognize problems to determine which questions were easy to understand, as confirmed by the interview results. At the stage of planning the solution, RA was able to devise problem-solving steps and answer questions according to the symbolic stage.

P: After reading all the questions, which question do you think is easier to understand and work on?

RA: Of all the questions, I think question no. 3 is easy to work on.

P: Why question no. 3?

RA: Because in question no. 3, I can understand and work on it using the Pythagorean theorem that we have learned.

3.	(Tahap Simbolik)
	$a. a^2 = b^2 + c^2$
	= 92+122
	= 91 + 144
	a²=225
	a = V225 = 15 cm
	b_1 $b^2 = C^2 + a^2$
	$=25^2+7^2$
	= 625 + 99
	=674
	b = V674 = 25,96 cm

Figure 11. Results of the Problem-Solving Ability Test for Subject RA Number 3

In solving problems, RA successfully completed questions 3a and 3b correctly using symbols or formulas without the aid of pictures, in accordance with the symbolic stage. In the evaluation stage, RA reviewed his answers by reading and matching them with the problems, in accordance with the confirmation of the interview results.

Problem-Solving Ability Test Results based on Bruner's Enactive Stage Theory on **Question Number 1 Subject MR**

In the problem identification indicator, MR read the question but did not understand the meaning of the question, as confirmed by the interview results. In the solution planning stage, MR was unable to devise problem-solving steps in accordance with the content of the question.

P : What was the first step you took to solve the problem?

MR: I read question no. 1, but I didn't know what to do because I wasn't allowed to use formulas.

P: After completing the question, did you reread your answers before submitting them? MR: No, sir, because I didn't answer that question.

In solving the problem, MR did not provide answers to questions 1a and 1b because of difficulty imagining the activities directly, so they were considered insufficient. At the evaluation stage, MR also did not check the answers or realize the mistakes, as confirmed by the interview results.

Problem-Solving Ability Test Results based on Bruner's Enactive Stage Theory on Question Number 2 Subject MR

In the problem identification indicator, the subject MR read the question but did not understand the meaning of the question, as confirmed by the interview results. In the solution planning stage, MR was unable to devise problem-solving steps in accordance with the content of the question.

P: What was the first step you took to solve the problem?

MR: I read the question given, sir, but I did not understand the meaning of this question, just like question 1.

In solving the problem, MR did not provide answers to questions 2a and 2b because of difficulty imagining the activities directly, so it was considered insufficient. At the evaluation stage, MR also did not check the answers or realize the mistakes, as confirmed by the interview results..

Problem-Solving Ability Test Results based on Bruner's Enactive Stage Theory on Question Number 3 Subject MR

On the indicator of identifying problems, MR subjects carefully read the questions to understand their meaning and determine which questions are easy to understand and can be answered. This is reinforced by the results of interviews with MR subjects.

P: After reading all the questions, which ones do you think are easier to understand and answer?

MR: Of all the questions, I think question no. 3 is easy to answer.

P: Why do you think question no. 3 is easy to answer?

MR: Because in question no. 3, I can use the Pythagorean theorem.

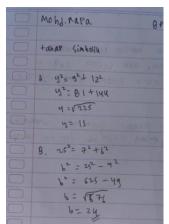


Figure 12. Results of the Problem-Solving Ability Test for Subject MR Number 3

At the planning stage, MR subjects were able to apply this in problem-solving tests, as demonstrated by their ability to answer questions at the symbolic stage. This occurred because MR had planned the steps to solve the problem in accordance with the intent of the question, as

confirmed by interviews. On the problem-solving indicator, the MR subject successfully answered questions 3a and 3b correctly in accordance with Bruner's symbolic stage theory, namely using symbols or formulas without the aid of pictures, as seen from the results of their answers. Furthermore, on the re-evaluation indicator, MR was able to draw conclusions, recheck the results of their answers, and match them with the problems given, which was also reinforced by the results of interviews with researchers.

Discussion

The findings of this study reveal that students exhibit uneven development across Bruner's representational stages when solving Pythagorean theorem problems. The data show that many students begin problem solving without relying on concrete or experiential understanding, indicating weaknesses in enactive representation. This suggests that early conceptual grounding through hands-on or physical modeling is not sufficiently established in their prior learning experiences. Bruner emphasized that enactive representation forms the foundation for later reasoning, and the limited performance at this stage reflects instructional environments where concrete exploration is rarely emphasized.

Performance at the iconic stage also varied. Only a small proportion of students demonstrated the ability to use diagrams or visual representations effectively. This difficulty aligns with Sukmawati & Rakhmawati (2023) argument that learners often struggle to coordinate visual and analytical processing because they have not yet developed the semiotic flexibility needed to interpret and transform visual information into mathematical relationships. The iconic stage should serve as a bridge between direct experience and symbolic abstraction, yet the results indicate that this bridging function is not fully realized in students' learning processes (Mailani, et al., 2022; Limbach & Waugh, 2020).

Despite weaknesses in the earlier stages, most students demonstrated stronger symbolic reasoning abilities. This indicates that symbolic manipulation has been prioritized in their instructional experiences, a pattern commonly reported in mathematics education research (Farid, 2024). Prior studies in Indonesian classrooms, for example, show that students are frequently trained to apply formulas procedurally without developing deeper conceptual foundations (Hatip & Setiawan, 2021). Consequently, symbolic proficiency may mask underlying conceptual fragility. The tendency to rely on formulas rather than understanding geometric relationships echoes the broader critique that mathematics instruction is often dominated by procedural fluency.

A noteworthy finding is the inconsistency between self-reported questionnaire results and actual performance in problem solving. These discrepancies imply that students may not accurately perceive their own thinking processes or that representational stages may not function as fixed categories (Astutiani & Hidayah, 2019).. Instead, they may be activated flexibly depending on task familiarity and the types of representations emphasized in classroom instruction. This supports Bruner's claim that representational development is experience-driven rather than tied to chronological age and aligns with evidence showing that learners may "skip" or underdevelop certain stages when instruction is unbalanced (Safitri et al., 2020).

Overall, the pattern of results (weak enactive and iconic performance but strong symbolic performance) indicates that students' representational development may be top-heavy, with symbolic skills advancing faster than conceptual grounding. This imbalance highlights the need for instructional approaches that explicitly integrate enactive and iconic representations before moving toward symbolic abstraction. Strengthening representational fluency across these stages is essential for developing deeper and more flexible mathematical understanding. The

findings have important implications for classroom practice. Learning activities involving manipulatives, geometric modeling, dynamic visualization tools, and structured representational transitions can help students develop more coherent conceptual frameworks. The results also reinforce the theoretical significance of Bruner's framework, demonstrating that representational stages remain relevant for explaining students' differential performance in mathematical problem solving.

Conclusion

The analysis of questionnaire responses, problem-solving tests, and interviews indicates that students exhibit varied thinking processes across Bruner's enactive, iconic, and symbolic stages. Although most students show adequate to strong performance at the symbolic stage, their overall representational development does not follow the ideal progression proposed by Bruner. Weaknesses in the enactive and iconic stages suggest that students often reach symbolic procedures without sufficient grounding in concrete experiences or visual reasoning. As a result, their symbolic abilities tend to be procedural rather than conceptually based, which limits the depth and flexibility of their mathematical understanding. These findings highlight the need for instructional designs that strengthen enactive and iconic representations before advancing to symbolic abstraction so that conceptual understanding can develop more coherently.

The implications of this study emphasize the importance of integrating all three of Bruner's representational stages into mathematics instruction. Teachers should design learning experiences that incorporate concrete manipulatives, visual media, and structured transitions toward symbolic reasoning, while also adapting strategies to accommodate differences in students' thinking processes. This approach supports the development of deeper conceptual understanding and reduces reliance on memorized procedures. Developers of curriculum and teaching materials should also include varied representational forms to better support students' cognitive development. Future research should employ larger samples and more diverse methodological approaches to examine the effectiveness of representation-based learning strategies in improving problem-solving abilities. Based on these findings, teachers are encouraged to avoid moving students directly to abstract symbol manipulation without first building understanding through concrete activities and visual representations. Attention to students' thinking processes can help identify appropriate strategies to enhance learning outcomes. This study is limited by its small sample size and exclusive use of qualitative methods; therefore, future research should expand the sample and consider quantitative or mixed-method approaches to strengthen generalizability and examine the impact of representation-based strategies more comprehensively.

Conflict of Interest

The authors declare that there is no conflict of interest.

Authors' Contributions

The first author, M.E.W., played a role in designing the research, creating the research instruments, understanding the research ideas, collecting data, analyzing data, processing data, and presenting the results and discussion of the research. The second author, R.T., participated in reviewing the research and adjusting the overall information in the research. The third author, D.I., participated in adjusting the research and discussion as well as finalizing the results of the work. The total contribution to the conceptualization, writing, and correction of this article is as follows: M.E.W.: 50%, R.T.: 25%, and D.I: 25%.

Data Availability Statement

The authors state that the data supporting the findings of this study will be made available by the corresponding author, [M.E.W.], upon reasonable request.

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