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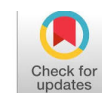
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Designing a Pre-Calculus Learning Module for Pre-Service Mathematics Teachers through the Hypothetical Learning Trajectory Approach

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ABSTRACT

The study identified several learning obstacles (LO) experienced by students in Calculus I, particularly misconceptions related to algebraic concepts, graphical representations of functions, absolute values, inequalities, exponents, logarithms, and trigonometry. These obstacles negatively affect students' conceptual readiness for learning Calculus. This study aims to develop an HLT-based pre-calculus module for 41 prospective mathematics teachers at a state university in Indonesia. The instruments included a diagnostic test, interview guidelines, and expert validation forms for the module. Data were collected through literature review, analysis of learning obstacles based on diagnostic test and interview results, and expert validation of the module. The pre-calculus module was developed using the 4D model (Define, Design, Develop, Disseminate), and the data were analyzed using a descriptive qualitative approach. Expert validation results indicated that the module's content, language, and presentation achieved validity scores above 80% across all aspects. Considering the effectiveness of the HLT framework, the module is expected to support scaffolding, reduce misconceptions, and map students' conceptual thinking trajectories. Furthermore, it is expected to serve as an independent learning resource that supports a logical transition from algebraic prerequisites to Calculus I concepts.



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Introduction

Calculus is a fundamental course that serves as the foundation for understanding various advanced subjects such as differential equations, real analysis, and other mathematics courses (Hajizah & Salsabila, 2024). Calculus also plays a central role as an essential branch of mathematics with wide applications in science, economics, and engineering (Septiani & Harisman, 2025). Although students often perceive calculus as a difficult subject, its concepts

are frequently used to solve real-world problems that cannot be addressed through elementary algebra (Hajizah & Salsabila, 2024; Khasanah & Febriana, 2024). Mastery of calculus is therefore crucial for the development of students' mathematical abilities. Consequently, the Calculus course is positioned as one of the core foundational courses in the curriculum for prospective mathematics teachers. Students' mastery of these fundamental concepts not only determines their success in subsequent courses but also shapes their readiness as future mathematics educators.

Despite its importance, numerous studies have reported that students continue to face difficulties in understanding calculus concepts. Research has shown that some students are unable to connect the definition of a function with its limit value (Sulastri et al., 2022). Students' inability to apply prior knowledge, such as algebraic factorization, may hinder their capacity to solve more complex calculus problems (Ismail & Bempah, 2018). Other findings indicate that students experience limitations in understanding composite functions, which subsequently cause learning obstacles in the chain rule topic (Puspita et al., 2022). These challenges are not limited to technical skills but also involve conceptual understanding and mathematical thinking processes. Many students struggle with limit and differential problems due to their limited ability to solve rational inequalities (Mirza et al., 2022). Even pre-service mathematics teachers encounter similar obstacles and difficulties in learning Calculus (Susilowati, 2021; Musyrfah et al., 2022).

Weak understanding of fundamental concepts often leads to difficulties in learning derivative and integral concepts in calculus courses (Ramadanti & Pujiastuti, 2020). Some studies have also identified challenges in solving integral problems related to students' understanding of the natural logarithm concept (Nurhayati & Ratnaningsih, 2023). Hakiki et al. (2025) further revealed that students struggle to comprehend abstract concepts and face difficulties in solving problem-solving and higher-order thinking tasks. These findings indicate that some students have not yet mastered essential prerequisite concepts needed before studying calculus. Given these identified difficulties in learning calculus, particularly those related to prerequisite concepts, strengthening students' understanding of foundational topics is necessary. Hajizah & Salsabila (2024) highlight several important pre-calculus topics, including the real number system, inequalities, absolute values, and functions, which serve as preliminary materials before Differential Calculus. Strengthening pre-calculus content is therefore essential to ensure students have a solid foundation and that the learning process can proceed effectively.

To design effective learning, it is crucial to consider the Hypothetical Learning Trajectory (HLT) framework. According to Arnellis et al. (2019), HLT functions as "a bridge between instructional theory and classroom learning practice." HLT consists of three main components: learning goals, a sequence of learning activities, and hypotheses of students' learning processes (Apriyanti et al., 2019). The implementation of HLT also emphasizes the importance of appropriate scaffolding. This aligns with the findings of Nurhayati et al. (2023), which show that students who received scaffolding were able to successfully solve the given problems. Using HLT as a pedagogical guide helps students overcome learning obstacles (Apriyanti et al., 2019). Through HLT, curriculum designers can predict students' learning responses and anticipate potential difficulties, making the learning process more structured and effective. This approach has been widely used in the development of mathematics learning across different levels to help students construct concepts through meaningful and sequential processes (Daro et al., 2011).

In Indonesia, various studies have adapted the Learning Trajectory (LT) approach in the development of mathematics instructional materials. Muhtadi et al. (2024) developed an HLT for the topic of exponents and demonstrated that a well-designed learning trajectory can

significantly improve students' conceptual understanding. Patmala et al. (2025) designed an HLT on logarithms based on the context of Realistic Mathematics Education (RME) and successfully guided students in constructing the meaning of logarithms through exploratory activities. Meanwhile, Hamdani et al. (2023) integrated the LT approach with local cultural contexts (specifically Jambi Malay ethnomathematics) to facilitate the understanding of trigonometric concepts. These studies demonstrate that learning trajectories designed according to students' characteristics and local contexts can enhance learning effectiveness.

Despite the extensive application of HLT in various mathematics topics, no studies have yet focused on developing a Pre-Calculus module that systematically considers students' learning trajectories. Therefore, this research aims to develop a pre-calculus module constructed based on the HLT framework tailored to students' needs. Through this module, students are expected to avoid misconceptions, strengthen their readiness for studying calculus, and develop into reflective and systematic future mathematics educators. This study also utilizes findings of students' conceptual errors in calculus as the foundation for constructing the HLT, ensuring that the developed module is not only theoretically grounded but also responsive to students' real learning needs and challenges. Consequently, this study is expected to contribute to the development of a holistic, contextual, and didactically grounded learning design based on the learning trajectory framework.

Method

Research Type

This study aims to develop a pre-calculus module based on the Hypothetical Learning Trajectory (HLT) framework. The HLT serves as a strategic approach that ensures learning focuses not only on the final outcomes but also on students' thinking trajectories. Through the development of this module, students are expected to avoid misconceptions, enhance their readiness for learning Calculus I, and develop into reflective and systematic future mathematics educators. The research employed a Research and Development (R&D) design in which the pre-calculus module was developed in several stages and empirically tested with undergraduate students enrolled in the Mathematics Education Study Program at a state university in Indonesia who were taking Calculus I. This design was chosen because it allows for an iterative, flexible, and adaptive process that results in a validated and effective product capable of addressing students' learning obstacles (LOs). To ensure the quality, comprehensibility, and potential effectiveness of the developed module, the 4D development model (Define, Design, Develop, and Disseminate) was adopted. The Define stage involved identifying and analyzing the learning problems to be addressed, the Design stage focused on developing the HLT as the conceptual foundation for the module, the Develop stage involved constructing and validating the module through expert review, and the Disseminate stage consisted of distributing the finalized module to students for implementation.

Participants and Instruments

The participants of this study were 41 first-semester students enrolled in the Mathematics Education Study Program at a state university in Indonesia who were taking Calculus I. Several instruments were employed, including a diagnostic test, an interview guide, and a module validation sheet. The diagnostic test was used to identify and analyze students' learning obstacles in Calculus I. The interview guide served as a framework for conducting interviews with selected students to confirm and explore their reasoning in solving diagnostic test items,

thereby providing accurate and detailed information regarding their learning difficulties. The module validation sheet was used to assess the content, language, and presentation quality of the module, and the feedback obtained from expert validators (lecturers specialized in Calculus) was used to refine and improve the module to ensure its academic and pedagogical quality.

Data Collecting and Analysis

Data collection procedure also followed the 4D model. In the Define stage, the researcher identified students' learning needs through the analysis of learning obstacles in Calculus I, supported by a literature review. Students' written responses to the diagnostic test were analyzed, and interviews were conducted to clarify ambiguous or incomplete responses. In the Design stage, the researcher developed a Hypothetical Learning Trajectory (HLT) tailored to students' needs, which became the basis for constructing the pre-calculus module. During the Develop stage, the module was created according to the designed HLT and subsequently validated by expert reviewers to evaluate its content, language clarity, and visual presentation. Revisions were made based on their feedback. In the Disseminate stage, the revised module was distributed to students enrolled in Calculus I for further use and evaluation. The collected data were analyzed using a descriptive qualitative approach, focusing on expert validation results and students' responses to the module's use. The analysis aimed to determine how the developed module facilitated students' learning and helped them overcome identified learning obstacles.

Results and Discussion

Define Phase

The Define phase in this study was conducted to produce an instructional module that meets the learning needs of pre-service mathematics teachers. A needs analysis was therefore carried out as the foundation for identifying learning problems. This analysis included curriculum analysis, content analysis, and an analysis of students' learning obstacles (LOs). To gather data for the needs analysis, the researcher conducted interviews and observations with students. During the curriculum and content analysis process, the researcher collaborated with Calculus course lecturers to validate the findings obtained from the field.

One of the key components of the needs analysis was the analysis of learning obstacles. In the initial stage of this process, the researcher developed a diagnostic test instrument specifically designed to identify learning obstacles related to fundamental calculus (pre-calculus) concepts such as limits, derivatives, integrals, and the interpretation of function graphs. The diagnostic test was administered to pre-service mathematics teachers, and to strengthen the findings from the test analysis, follow-up interviews were conducted with several students who showed misconceptions in their understanding of basic calculus concepts. Based on the results of the diagnostic tests and interviews, the researcher identified several types of learning obstacles, including epistemological obstacles and ontogenical obstacles experienced by students.

Figure 1 presents a student's response related to the concept of absolute value. The student made an error in defining the given absolute value function and in determining the corresponding interval, which led to an incorrect final answer. Through interviews, it was revealed that the student was confused when required to define the given function but was able to define simpler absolute value functions such as $|x|$.

$$f(x) = \begin{cases} \frac{|x-1|}{x-1}, & 0 < x < 1 \\ 4x-3, & 1 \leq x < 2 \\ x^2+1, & x \geq 2 \end{cases} \rightarrow \begin{cases} \frac{x-1}{x-1}, & x > 0 \\ \frac{-x+1}{x-1}, & x < 0 \end{cases}$$

Figure 1. Error in Understanding the Concept of Absolute Value

The response shown in Figure 2 was given by another student, who made an error in performing fraction operations. During the interview, the student quickly realized the mistake and was able to provide the correct solution.

$$\begin{aligned} \sqrt[2]{50} &= \sqrt[2]{99+1} \Rightarrow x = 99, \Delta x = 1 \\ f(x) &= \sqrt{x} \\ f'(x) &= \frac{1}{2\sqrt{x}} \\ \Delta y &= f(x + \Delta x) \\ &= f(x) + f'(x) \cdot \Delta x \\ &= \sqrt{99} + \frac{1}{2\sqrt{99}} \cdot 1 \\ &= 7 + \frac{1}{2 \cdot 7} \cdot 1 \\ &= 7 + \frac{1}{14} = 7.07 \end{aligned}$$

Figure 2. Error in Performing Fraction Operations

The finding illustrated in Figure 3 represents an error in algebraic computation, where the student made a mistake in multiplying algebraic expressions.

$$\begin{aligned} 3. a) \tan^{-1}(\cos x^3 + (x^2y + y^4)^5) &= 10 \\ 0) \frac{1}{1 + (\cos x^3 + (x^2y + y^4)^5)^2} \cdot (-\sin x^3) \cdot 3x^2 + 5(x^2y + y^4)^4 \cdot (2xy + x^2 \frac{dy}{dx} + 4y^3 \frac{dy}{dx}) &= 0 \\ 0) \frac{-3x^2 \sin x^3}{1 + \cos^2 x^3} + 5(x^2y + y^4)^4 \cdot (2xy + x^2 \frac{dy}{dx} + 4y^3 \frac{dy}{dx}) &= 0 \\ 0) \frac{-3x^2 \sin x^3}{1 + \cos^2 x^3} + 5(x^2y + y^4)^4 \cdot 2xy &= x^2 \frac{dy}{dx} + 4y^3 \frac{dy}{dx} \\ \frac{dy}{dx} &= \frac{-3x^2 \sin x^3 + (5x^2y + 5y^4)^4 \cdot 2xy}{1 + \cos^2 x^3 + (x^2 + 4y^3) \cos^2 x^3} \end{aligned}$$

Figure 3. Error in Performing Algebraic Operations

Figure 4 shows another student's error in factoring. The student incorrectly identified the algebraic form to be factored, mistaking it for a difference of two squares and proceeding to factorize it as such.

$$\lim_{x \rightarrow 5^+} \frac{x-5}{\sqrt{x^2-25}}$$

$$\lim_{x \rightarrow 5^+} \frac{x-5}{\sqrt{(x-5)(x+5)}}$$

$$\lim_{x \rightarrow 5^+} \frac{x-5}{\sqrt{x-5} \sqrt{x+5}}$$

$$\lim_{x \rightarrow 5^+} \frac{x-5}{\sqrt{x-5} \sqrt{x+5}}$$

$$\lim_{x \rightarrow 5^+} \frac{x-5}{\sqrt{x-5} \sqrt{x+5}}$$

$$\lim_{x \rightarrow 5^+} \frac{\sqrt{x-5}}{\sqrt{x-5} \sqrt{x+5}} \rightarrow \frac{0}{0} \text{ bil real}$$

maka nilai limit = 0

Figure 4. Error in Factoring Algebraic Expressions

In Figure 5, a student incorrectly stated that the indeterminate form $0/0$ equals 1. The student explained that this result was obtained because both the numerator and denominator were equal, failing to recognize that $0/0$ is an indeterminate form.

$$\lim_{x \rightarrow 1^-} \frac{x-1}{x-1} = 1$$

$$\lim_{x \rightarrow 1^+} \frac{4x-3}{x-1} = 1$$

$$\lim_{x \rightarrow 1^-} \frac{x-1}{x-1} = \lim_{x \rightarrow 1^+} \frac{4x-3}{x-1}$$

Figure 5. Error in Interpreting $0/0$ as Equal to 1

Another error, shown in Figure 6, was made by a student who struggled with trigonometric transformations. The student made an error when attempting to convert a trigonometric expression into an equivalent form.

$$\lim_{x \rightarrow 0} \frac{\cos^2 2x - 1}{4x^2} = \lim_{x \rightarrow 0} \frac{1 + 2 \sin^2 2x}{4x^2} = 1 + \lim_{x \rightarrow 0} \frac{2 \sin 2x \cdot \sin 2x \cdot 4}{2x \cdot 2x} = 1 + 1 \cdot 4 = 1 + 2 = 3$$

Figure 6. Error in Transforming Trigonometric Forms

These observed errors in students' responses clearly indicate the presence of learning obstacles in the learning process. This finding is consistent with the results reported by Mirza et al. (2022) and Hakiki et al. (2025), who also found that students experience significant learning barriers in understanding fundamental calculus concepts.

Design Phase

The Design phase was the follow-up to the findings obtained from the needs analysis. Within the framework of the Theory of Didactical Situations (Brousseau, 2002), the analysis of learning obstacles (LOs) served as the foundation for constructing the Hypothetical Learning Trajectory (HLT), which was subsequently used in the development of the pre-calculus module.

Based on the results of the LO analysis and student interviews, the HLT for pre-calculus material is illustrated in Figure 7.

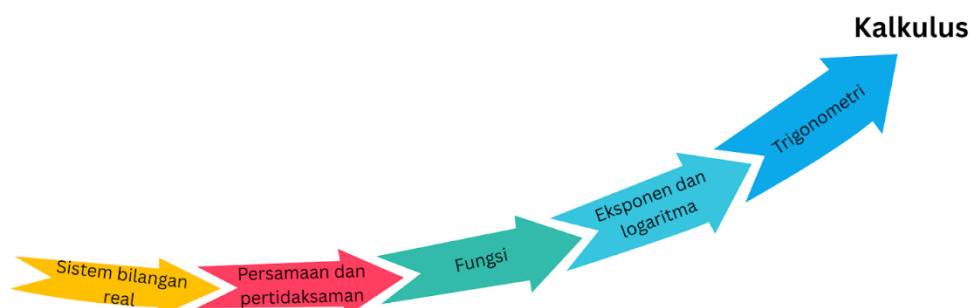


Figure 7. Hypothetical Learning Trajectory

The results of the LO analysis and interviews suggest that students need reinforcement in understanding the real number system, arithmetic operations, and the concepts of equations and inequalities. Following this, students require further strengthening in topics related to relations and functions, as well as various types of functions and their corresponding graphs, particularly the absolute value function. After mastering these, students are expected to deepen their understanding of exponents, logarithms, and trigonometry, including their properties. By reinforcing these foundational mathematical concepts, students are expected to be better prepared and more capable of engaging effectively in Calculus I learning activities.

Develop Phase

The Develop phase in this study aimed to design a pre-calculus instructional module. The process began with the selection of topics to be included in the module. In preparing the content, the researcher collaborated with the Calculus I teaching team, taking into account the Hypothetical Learning Trajectory (HLT) that students experience. Consistent with the study conducted by Hajizah & Salsabila (2024), the topics agreed upon for inclusion in the module were the real number system, inequalities, relations and functions, absolute value, linear equations, exponents and logarithms, and trigonometry.

The learning objectives of the developed module were designed to equip students with a solid foundation for understanding and applying mathematical concepts as a preparation for Calculus I. Specifically, the objectives are threefold. First, students are expected to understand fundamental mathematical concepts that serve as an introduction to calculus, including the ability to explain the concepts of relations, functions, and graphs, as well as to comprehend the properties of algebraic, trigonometric, exponential, and logarithmic functions. Second, students should be able to apply mathematical operations effectively, such as performing algebraic operations on functions (covering addition, subtraction, multiplication, division, and function composition) and utilizing the properties of exponents and logarithms to solve various equations. Finally, the module aims to prepare students for the Calculus I course by ensuring mastery of essential mathematical concepts that form the basis for understanding calculus and other subsequent, related courses. Through these learning objectives, students are expected to develop a strong foundational understanding and to be better prepared to engage effectively in learning activities, particularly in the Calculus I course. After the module was developed, it underwent expert validation by the Calculus teaching team. The validators assessed the content validity, language appropriateness, and presentation quality of the designed module. Several revisions were made based on the validators' feedback to ensure that the module met high academic and pedagogical standards.

Table 1. Results of Module Validation

No	Aspect Assessed	Indicator	Percentage of Validator Scores
1	Content Feasibility	The material presented corresponds to the topic discussed	100%
		The learning objectives align with the basic competencies or learning outcomes	100%
		The module structure is well-organized and follows a logical sequence	100%
		The evaluation component aligns with the intended competencies	50% / 50%
		The module supports students' independent learning	100%
2	Language	The language used adheres to grammatical conventions	100%
		The language is clear and free from ambiguity	100%
		The language matches students' cognitive development level	100%
3	Presentation	The visual layout and design of the module facilitate students' understanding of the content	100%

The validation results indicate that the developed module is of high quality in terms of content accuracy, linguistic clarity, and presentation design, with only minor revisions required in the evaluation component. This validated module is therefore considered ready for implementation and further testing in classroom settings.

Disseminate Phase

The Disseminate phase represents the final stage in the development process of the pre-calculus instructional module. This stage aimed to distribute the research product (the pre-calculus learning module) to first-year students enrolled in the Calculus I course. After the module was validated and approved by the Calculus teaching team, it was uploaded to the Learning Management System (LMS) to ensure easy access for students taking the course.

This dissemination phase serves as a strategic step to ensure that students receive instructional materials specifically designed and adapted to their learning needs, particularly in addressing the previously identified learning obstacles (LOs). This approach distinguishes the present study from previous research. For instance, earlier studies such as [Abadi & Hidayanti \(2020\)](#) focused primarily on the design of interactive digital modules, whereas this study emphasizes the development of a digital module based on the Hypothetical Learning Trajectory (HLT), which was grounded in a thorough analysis of students' learning obstacles. Therefore, this research contributes a novel perspective to the development of adaptive and contextually relevant instructional designs for Calculus I learning.

Conclusion

This study, which employed the 4D (Define, Design, Develop, Disseminate) model, successfully produced a pre-calculus instructional module developed using the Hypothetical Learning Trajectory (HLT) approach. The Define stage focused on analyzing students' learning needs through several forms of analysis. The identification of learning obstacles served as the foundation for designing the HLT, with these obstacles being identified through diagnostic tests and interviews. The Design stage involved constructing the HLT diagram that represents students' thinking trajectories. The Develop stage focused on creating the instructional module based on the constructed HLT, while the Disseminate stage served as the final phase, in which information about the HLT-based pre-calculus module was distributed to students. Based on the findings and discussion presented, it can be concluded that the developed HLT-based pre-calculus module is valid, practical, and has strong potential to enhance the quality of learning in

the Calculus I course for pre-service mathematics teachers. Thus, the development of this HLT-based module contributes to the didactical process by facilitating students' conceptual understanding in a more structured manner. This study was developed based on the characteristics of a specific student group; therefore, the module may not yield optimal results if applied to students with different characteristics. Further research is recommended to extend and adapt the pre-calculus module so that it can be effectively implemented across diverse student populations.

Conflict of Interest

The authors declare that there is no conflict of interest.

Authors' Contributions

U.N.A. developed the main research ideas, collected the data, and conducted the discussion of the findings. The other two authors (N.U. and N.S.P.) actively participated in the development of the theoretical framework, research methodology, data organization, data analysis, and approval of the final version of the manuscript. All authors have read and approved the final version of this paper. The total percentage of contributions for the conceptualization, preparation, and revision of this manuscript is as follows: U.N.A.: 40%, N.U.: 30%, and N.S.P.: 30%.

Data Availability Statement

The authors declare that data sharing is not applicable to this article, as no new data were created or analyzed in this study.

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