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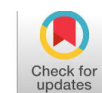
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Enhancing Students' Mathematics Learning Outcomes through Problem-Based Learning

Agnes Pesta Kristiani Pardede^{1*}, Philotheus E.A Tuerah² , Ermita³ , Galih Albarra Shidiq⁴

^{1,2,3}Department of Mathematics Education, Faculty of Mathematics, Natural Sciences, and Earth Sciences, Universitas Negeri Manado

⁴Department of Educational Technology and Communications, Faculty of Education, Chulalongkorn University

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ABSTRACT

Low levels of student engagement and learning motivation have led to limited critical thinking skills, which in turn have affected students' learning outcomes that remain below the Minimum Mastery Criteria (MMC). This study aims to determine whether students who received instruction through the Problem-Based Learning (PBL) model achieved higher mathematics learning outcomes than those taught using the conventional (lecture-based) model on the topic of Relations and Functions. This research employed a quasi-experimental method with a pretest-posttest control group design. Population consisted of eighth-grade students from SMP Negeri 3 Tondano, while the samples were class VIII-A (22 students) as the experimental group and class VIII-B (17 students) as the control group. The research instrument comprised written essay tests administered as pretests and posttests in both groups. Data were analyzed using inferential statistical procedures. Data were first tested for normality using the Liliefors test, followed by a homogeneity test (F-test). After both assumptions were met, an independent samples t-test (two-tailed) was performed to test the hypothesis. The results showed a statistically significant difference in mathematics learning outcomes between students who learned through the PBL model and those taught using the conventional lecture-based method. Therefore, it can be concluded that the implementation of the PBL model effectively enhances students' mathematics learning outcomes on the topic of Relations and Functions, providing evidence that active, problem-oriented instruction can improve students' conceptual understanding and achievement in mathematics.



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Corresponding Author:

Agnes Pesta Kristiani Pardede,
Department of Mathematics Education,
Faculty of Mathematics, Natural Sciences, and Earth Sciences,
Universitas Negeri Manado
Jl. Kampus Unima, Tonsaru, South Tondano District, Minahasa Regency, North Sulawesi 95618, Indonesia
Email: agnespestakristiani00@gmail.com

Introduction

Globally, improving students' engagement and higher-order thinking skills in mathematics has become a central concern in educational research (Habsyi et al., 2022; Nur, 2024; Rahayuningsih et al., 2022). International reports, including those from the OECD and UNESCO, have emphasized that mathematics education should not merely focus on procedural fluency but also on cultivating creativity, critical thinking, and problem-solving abilities to prepare students for the demands of the twenty-first century (OECD, 2018). Education plays a vital role in the progress of a nation, serving as a key driver for the development of high-quality human resources. Accordingly, the advancement of a nation greatly depends on an education system capable of producing intelligent, skilled, and competitive generations. One of the main efforts to improve the quality of education today is the implementation of effective classroom teaching methods, particularly through the use of innovative learning models (Hollebrands & Lee, 2020; Mohamed Abdul-Rahmana, 2020; Putranto & Marsigit, 2018). A well-designed learning model can make the learning process more engaging and varied, thereby increasing its effectiveness. In mathematics education in particular, distinctive approaches are required to help students better understand abstract mathematical concepts.

Mathematics is a universal discipline that plays a pivotal role across various domains, particularly in developing students' cognitive abilities. It trains learners to think critically, reason logically, and solve problems effectively. Mathematics also cultivates structured thinking, perseverance, and accuracy (Callingham & Siemon, 2021; Eriksson & Sumpter, 2021), as well as computation, problem-solving, and reasoning skills (Amador et al., 2024; Jones & Kuster, 2021; Tallman & Frank, 2020). However, many students still perceive mathematics as difficult and uninteresting. This perception is influenced by how teachers create learning environments that either stimulate or inhibit students' motivation and engagement in mathematics learning.

Observations at SMP Negeri 3 Tondano revealed several challenges in classroom practices. Low student engagement was one of the most pressing issues. Many students appeared hesitant to ask questions, express opinions, or participate actively during lessons. Instead, they tended to remain passive, merely taking notes without providing feedback or responses. Consequently, the learning process became monotonous and non-interactive, limiting students' opportunities to develop critical-thinking skills. This lack of engagement also negatively affected their comprehension of mathematical concepts. Quantitative data support these observations: based on the mathematics examination results of Grade VIII students in the 2023/2024 academic year, only 34 out of 81 students achieved the Minimum Mastery Criteria (MMC) score of 75, with an average of 76. Meanwhile, 47 students scored an average of 65.5, indicating that nearly 60% failed to meet the mastery threshold and experienced difficulties in understanding and solving mathematical problems.

The mathematics teacher at the school further reported that students often struggled to comprehend lessons, particularly those with weak mathematical foundations. Low learning motivation and the continued reliance on conventional, lecture-based teaching methods were identified as key factors hindering learning effectiveness (Tran et al., 2020). During lessons, students frequently became distracted and disengaged, resulting in a monotonous and less stimulating learning atmosphere. Such conditions not only diminished students' enthusiasm but also discouraged active participation, further reinforcing the notion that mathematics is a difficult and tedious subject (Ostermann et al., 2018). To address these issues, teachers must adopt innovative instructional models that promote active student involvement. One promising approach is Problem-Based Learning (PBL). From a constructivist perspective, PBL positions students as active constructors of knowledge through authentic problem contexts, thereby

fostering deep understanding and transferable skills in mathematics (Kotto et al., 2022; Lee et al., 2019; Nolaputra et al., 2018). PBL actively engages students in problem solving, thus enhancing understanding, creativity, and critical-thinking abilities. PBL model significantly improved mathematics learning outcomes among seventh-grade students on the topic of Social Arithmetic, with higher average scores compared to those taught using the Direct Instruction model (Kim et al., 2013; Son & Lee, 2021). PBL had a significant impact on improving students' mathematical critical-thinking skills, demonstrated that its implementation increased both learning motivation and achievement in mathematics.

Although the effectiveness of PBL has been widely explored, limited studies have examined its application to the topic of Relations and Functions among eighth-grade students characterized by low classroom participation. This gap highlights the need to further investigate how PBL can support active engagement and improve mathematics learning outcomes in such contexts. Accordingly, this study aims to examine the impact of implementing the Problem-Based Learning (PBL) model on students' mathematics learning outcomes in the topic of Relations and Functions at SMP Negeri 3 Tondano. The study is expected to contribute to the body of knowledge in mathematics education by providing empirical evidence on how the PBL model can enhance engagement and achievement in low-participation learning contexts, particularly within Indonesian secondary schools.

Method

Research Design

This study employed a quantitative experimental approach aimed at examining the extent to which a specific treatment influences the research subjects. The research used a quasi-experimental design, as the participants were drawn from existing classroom groups rather than being randomly assigned at the individual level. The specific design implemented was the pretest–posttest control group design. In this design, two groups were selected randomly: one serving as the experimental group and the other as the control group. Both groups were given a pretest to measure their initial understanding and a posttest to assess their learning outcomes after the intervention. The experimental group received instruction using the Problem-Based Learning (PBL) model, while the control group was taught using the conventional lecture-based method. The same assessment instrument was administered to both groups to ensure comparability of results.

Table 1. Research Design

Group	Random	Pretest	Treatment	Posttest
Experimental Class	R	O ₁	X ₁	O ₂
Control Class	R	O ₃	X ₂	O ₄

Description:

R : Random selection of groups

O₁ : Pretest results of the experimental group

O₂ : Posttest results of the experimental group

O₃ : Pretest results of the control group

O₄ : Posttest results of the control group

X₁ : Treatment using the Problem-Based Learning (PBL) model

X₂ : Treatment using the conventional (lecture-based) learning model

Research Subjects

The population of this study comprised all eighth-grade students from SMP Negeri 3 Tondano, located at Jl. Sam Ratulangi No. 452, Tataaran I, Minahasa Regency, Indonesia. During the 2024–2025 academic year, the school consisted of two eighth-grade classes: Class VIII-A with 22 students and Class VIII-B with 17 students. Using a random sampling technique, the researchers assigned Class VIII-A as the experimental group and Class VIII-B as the control group. This sampling approach was chosen to ensure that both classes had an equal opportunity to be assigned to either condition. The experimental group received instruction using the Problem-Based Learning (PBL) model, whereas the control group was taught using the conventional lecture-based method. Both groups were taught by the same mathematics teacher to control potential variations in instructional delivery. Data collection was conducted in accordance with standard ethical research procedures and approved by the school administration.

Instruments

This study employed an essay-type test instrument that had been previously validated. The test items were reviewed and approved by expert validators before being administered to the students. The questions were developed based on indicators aligned with the lesson content and the specific treatment applied to each class. The validity and appropriateness of the instrument were ensured through expert validation. The validation process involved two experts: one lecturer from Manado State University and one mathematics teacher from SMP Negeri 3 Tondano. Prior to the final assessment, the validators provided several suggestions for revising and improving the instrument. After the revisions were made, the validators evaluated and assigned final validity scores to the instrument, confirming that it was suitable for use in the study.

Procedures

The research procedures were conducted in three main stages, namely preparation, implementation, and evaluation. In the preparation stage, classroom observations and interviews with the mathematics teacher were carried out to identify the learning context and students' needs. In addition, learning materials and research instruments were developed and validated to ensure their relevance to the curriculum and research objectives. In the implementation stage, the study was conducted in two classes: one designated as the experimental group and the other as the control group. The experimental group received instruction using the Problem-Based Learning (PBL) model, while the control group was taught through the conventional lecture-based method. Each class had a total learning duration of 2×40 minutes per session, conducted over four meetings. During this stage, students in the experimental group were engaged in problem-solving activities that encouraged active participation and critical thinking, whereas the control group followed a teacher-centered approach focusing on explanation and note-taking. The evaluation stage involved administering a posttest to both groups after the instructional interventions. The test results were analyzed using appropriate statistical techniques to determine the effectiveness of the Problem-Based Learning (PBL) model compared to the conventional teaching method.

Analysis

Data for this study were collected using an essay-type test consisting of pretest and posttest items. The instrument, which had been validated by experts, was designed to measure students' problem-solving abilities before and after the instructional intervention. The collected data were analyzed using parametric statistical methods. To determine whether the data were normally distributed, a normality test was conducted using the Liliefors test. If the data were found to be normally distributed, a homogeneity test using the F-test was then performed to verify that the variances of the two groups were equal. After both assumptions of normality and homogeneity were satisfied, a two-sample t-test was applied to test the research hypothesis. The hypotheses tested in this study were formulated as follows:

$$H_0: \mu_1 = \mu_2$$

$$H_1: \mu_1 > \mu_2$$

where:

μ_1 = the mean difference between pretest and posttest scores of students taught using the Problem-Based Learning (PBL) model;

μ_2 = the mean difference between pretest and posttest scores of students taught using the conventional (lecture-based) method.

Results

Decriptive Analysis

This study was conducted at SMP Negeri 3 Tondano during the second semester of the 2024/2025 academic year. Two classes participated in the research: Class VIII-A (22 students) as the experimental group taught using the Problem-Based Learning (PBL) approach, and Class VIII-B (17 students) as the control group taught through the conventional lecture-based method. Data were collected through pretests and posttests administered before and after the learning interventions to evaluate students' mathematics achievement. The descriptive statistics of the pretest and posttest scores for both groups are presented below.

Table 2. Descriptive Statistics of the Experimental Group

No	Descriptive Statistics	Pretest	Posttest
1	Total Score	581	1,895
2	Mean	25.82	85.71
3	Minimum Score	15	77
4	Maximum Score	40	95
5	Standard Deviation	27.10	22.86
6	Variance	41.78	24.89

As shown in Table 2, the experimental group consisted of 22 students. The pretest results revealed a mean score of 25.82 with a standard deviation of 27.10 and a variance of 41.78, while the posttest results showed a mean score of 85.71 with a standard deviation of 22.86 and a variance of 24.89. These findings indicate a substantial increase in students' mathematics achievement after the implementation of the PBL approach.

Table 3. Descriptive Statistics of the Control Group

No	Descriptive Statistics	Pretest	Posttest
1	Total Score	441	1,351
2	Mean	25.94	79.47
3	Minimum Score	18	72

4	Maximum Score	38	90
5	Standard Deviation	5.92	6.62
6	Variance	35.06	43.76

As shown in Table 3, the control group consisted of 17 students. The pretest results indicated a mean score of 25.94, with a standard deviation of 5.92 and a variance of 35.06, whereas the posttest results showed a mean score of 79.47, with a standard deviation of 6.62 and a variance of 43.76. Although an improvement was observed in the control group, the increase was smaller than that observed in the experimental group.

Normality Test

A normality test was performed to verify whether the data were normally distributed. For the experimental group, the Liliefors test yielded a calculated value of $L_{calculated} = 0.056377$, which was lower than $L_{table} = 0.1840$, indicating that the data were normally distributed. Similarly, for the control group, $L_{calculated} = 0.11184 < L_{table} = 0.2071$, suggesting that the posttest scores were also normally distributed.

Table 4. Normality Test Results (Liliefors Test)

Group	$L_{calculated}$	L_{table}	Decision	Interpretation
Experimental	0.056	0.184	$L_{calculated} < L_{table}$	Data are normally distributed
Control	0.112	0.207	$L_{calculated} < L_{table}$	Data are normally distributed

Note. The Liliefors test was conducted at the 0.05 significance level. Results indicate that both the experimental and control groups met the assumption of normality.

Homogeneity Test

The homogeneity test was conducted to determine whether the variances of the two groups were equal. The results of the F-test showed that $F_{calculated} = 1.77222$, which was lower than $F_{table} = 2.15626$ at the 0.05 significance level. Therefore, the null hypothesis ($H_0: \sigma_1^2 = \sigma_2^2$) was accepted, indicating that the two samples had homogeneous variances.

Table 5. Homogeneity Test Results (F-Test)

Group Comparison	$F_{calculated}$	$F_{table} (\alpha = 0.05)$	Decision	Interpretation
Experimental vs. Control	1.772	2.156	$F_{calculated} < F_{table}$	Variances are homogeneous

Note. The F-test results confirmed that the variances between the experimental and control groups were homogeneous.

Hypothesis Testing

After confirming that the assumptions of normality and homogeneity were met, the independent samples t-test was conducted to test the research hypothesis using the posttest data. The results revealed that $t_{calculated} = 3.358$ and $t_{table} = 2.026$ at the 0.05 significance level ($\alpha = 0.05$). Since $t(37) = 3.358 > t_{table} = 2.026$, the null hypothesis ($H_0: \mu_1 = \mu_2$) was rejected, and the alternative hypothesis ($H_1: \mu_1 > \mu_2$) was accepted. The t-test results indicate a statistically significant difference in students' mathematics achievement between the experimental and control groups. Before the implementation of the PBL approach, students in the experimental group demonstrated relatively low understanding of mathematical concepts, with an average pretest score of 25.82, suggesting limited comprehension of the material. However, after the PBL intervention, their posttest mean score increased significantly to 85.71, reflecting a remarkable improvement in understanding and problem-solving ability.

Table 6. Hypothesis Test Results (Independent Samples t-Test)

Statistical Parameter	Experimental Group	Control Group	$t_{\text{calculated}}$	$t_{\text{table}} (\alpha = 0.05, df = 37)$	p-value	Decision
Posttest Mean	85.71	79.47	3.358	2.026	< .05	Reject H_0

Note. The independent samples t-test revealed a statistically significant difference between the two groups, $t(37) = 3.36$, $p < .05$, indicating that the Problem-Based Learning (PBL) approach led to higher mathematics achievement compared to the conventional lecture-based method.

In contrast, although students in the control group also experienced some improvement, the increase was not as substantial. The posttest mean score of the experimental group was notably higher than that of the control group, confirming the effectiveness of the PBL approach in enhancing learning outcomes. The difference between the two groups was statistically significant and not due to random variation ($t(37) = 3.358$, $p < .05$). This finding provides strong evidence that the Problem-Based Learning (PBL) approach is more effective than the conventional lecture-based method in improving students' mathematics achievement.

Discussion

The results of this study demonstrate that the Problem-Based Learning (PBL) approach significantly improves students' mathematics learning outcomes compared to the conventional lecture-based method. This finding aligns with several previous studies that have reported similar results. Lee et al. (2019) found that PBL had a significant positive effect on students' mathematical reasoning and problem-solving skills. Likewise, Makitalentu (2023) emphasized the superior effectiveness of PBL in mathematics instruction, particularly in teaching the topic of Relations and Functions. Aries (2022) also confirmed that PBL exerts a positive influence on mathematics learning outcomes, while Kotto et al. (2022) highlighted that the model produces a substantial impact on overall student achievement. Collectively, these studies support the conclusion that the implementation of PBL enhances both cognitive and affective dimensions of mathematics learning.

The improvement observed in this study can be explained through the principles of constructivist learning theory, which suggests that students construct knowledge actively through inquiry, collaboration, and reflection (Fuentealba et al., 2017; Hackenberg et al., 2021; Wilkie, 2020). In the PBL environment, students are encouraged to explore mathematical problems, test hypotheses, and discuss their reasoning with peers. This process enables them to connect abstract mathematical concepts to real-world situations, fostering meaningful understanding and critical thinking (Achmetli et al., 2019; Jones, 2017). In contrast, the conventional lecture-based method limits students' engagement, as learning remains teacher-centered and procedural, resulting in less opportunity for students to internalize concepts deeply.

In comparison to previous research, this study offers several novel contributions. First, the participants consisted of students with low learning participation and heterogeneous backgrounds, providing insights into PBL's effectiveness under more challenging and realistic classroom conditions. Second, the test instruments used in this study were developed to separately measure conceptual understanding and problem-solving skills, allowing for a more precise analysis of learning improvements in both domains. Third, the results revealed a distinct pattern of progress: the highest gain occurred in problem-solving and contextual reasoning indicators, which increased by approximately 72%, whereas conceptual understanding of Relations and Functions improved by about 60%. Such detailed differentiation between conceptual and procedural gains has not been explicitly reported in prior studies. Finally, despite the limited instructional time (only two sessions for one topic) the findings

confirmed that PBL remained effective, demonstrating its adaptability and efficiency in short-term learning contexts.

These results carry important theoretical and pedagogical implications. Theoretically, they reinforce the constructivist premise that problem-based inquiry fosters deeper cognitive processing, bridging the gap between mathematical abstraction and contextual application. Pedagogically, they show that PBL promotes active participation, motivation, and collaboration, skills essential for 21st-century learning (Viirman, 2015). For schools with diverse student populations and limited instructional time, PBL provides a flexible yet impactful framework to enhance learning outcomes. Therefore, this study not only supports existing evidence but also extends it by demonstrating the robustness of the PBL model when applied to heterogeneous learners and time-constrained mathematics instruction.

The learning dynamics observed during the PBL sessions suggest that students' cognitive engagement progressed through stages of exploration, connection, and abstraction. This process aligns with the levels of mathematical thinking proposed by Carlson & Thompson (2017), where learners initially make sense of problems through contextual representation before forming relational understanding. When students were encouraged to question, hypothesize, and justify their reasoning, they shifted from surface-level learning to deeper mathematical reflection. This transformation was especially evident in the experimental group, where collaborative discourse and peer explanations facilitated the internalization of key mathematical relationships between "relation" and "function." Thus, the effectiveness of PBL in this study is not only statistical but also conceptual, it builds structural understanding rather than procedural repetition.

From a cognitive-developmental perspective, the success of PBL can also be linked to students' metacognitive regulation during problem solving. The model's open-ended questioning and peer negotiation stimulated self-monitoring and self-evaluation—two critical components of metacognitive awareness. These processes enabled students to identify misconceptions, test multiple strategies, and evaluate their own reasoning paths. Such findings are consistent with the views of Degrande et al. (2017), who emphasize that effective problem-based tasks cultivate reflective control over one's thinking processes. Consequently, this study strengthens the argument that PBL not only enhances outcomes quantitatively but also transforms students' mathematical habits of mind qualitatively.

At the practical level, these findings carry significant implications for curriculum design and teacher professional development. For curriculum designers, the evidence underscores the importance of embedding real-world problem contexts and collaborative inquiry tasks into mathematics syllabi. For teachers, implementing PBL requires a pedagogical shift, from direct instruction toward facilitating dialogue, reflection, and scaffolding of student reasoning. Training programs should therefore equip teachers with skills in questioning techniques, formative assessment, and the orchestration of student collaboration. As noted by Hollebrands & Lee (2020), the effectiveness of student-centered learning models largely depends on teachers' ability to balance structure and autonomy within the classroom. The present findings thus offer a concrete example of how pedagogical innovation can thrive even in resource-limited and time-constrained school settings.

Conclusion

The findings of this study demonstrate that the Problem-Based Learning (PBL) model has a significant positive impact on students' mathematics learning outcomes, particularly in the topic of Relations and Functions. Students who learned through the PBL model showed greater conceptual understanding and problem-solving ability compared to those taught using

conventional lecture-based methods. These results confirm that engaging students in authentic problem-solving activities enables them to construct knowledge more meaningfully and to develop deeper comprehension of mathematical concepts. Furthermore, the study highlights that the PBL approach fosters students' active participation, critical thinking, and collaborative learning, which are essential skills for success in 21st-century education. The findings suggest that when properly implemented, PBL can serve as an effective pedagogical strategy to enhance learning motivation, classroom interaction, and long-term understanding of mathematics. Compared to conventional instruction, PBL encourages a shift from teacher-centered to student-centered learning, allowing students to take ownership of their learning process.

However, this study is limited by the use of a single essay-based test instrument, which focused only on cognitive outcomes and did not assess affective or collaborative dimensions of learning. Future research should therefore integrate a variety of instruments (such as observation checklists, attitude scales, and peer-assessment tools) to capture a more comprehensive picture of students' cognitive, affective, and social development in PBL-based mathematics learning. Overall, this study contributes to the growing body of evidence supporting the integration of student-centered, inquiry-based approaches such as PBL to promote meaningful and sustainable learning in mathematics education.

Conflict of Interest

The authors declare that there is no conflict of interest.

Authors' Contributions

The first author, A.P.K.P, contributed to data collection, research instrument preparation, instrument validation, theoretical framework development, methodology design, data organization and analysis, as well as interpretation and discussion of the findings. The second author, P.E.A.T, and the third author, E, participated in reviewing and revising the manuscript, and all authors have read and approved the final version of this paper. The percentage of contributions to the conceptualization, drafting, and revision of this manuscript is as follows: A.P.K.P.: (50%), P.E.A.T.: (20%), E.: (20%), and G.A.S.: 10%.

Data Availability Statement

The authors declare that the data supporting the findings of this study will be made available by the corresponding author, [A.P.K.P.], upon reasonable request.

References

- Achmetli, K., Schukajlow, S., & Rakoczy, K. (2019). Multiple Solutions for Real-World Problems, Experience of Competence and Students' Procedural and Conceptual Knowledge. *International Journal of Science and Mathematics Education*, 17(8), 1605–1625. <https://doi.org/10.1007/s10763-018-9936-5>
- Amador, J. M., Glassmeyer, D., & Brakoniecki, A. (2024). Teachers' noticing of proportional reasoning. *Journal of Mathematics Teacher Education*, January. <https://doi.org/10.1007/s10857-024-09625-7>
- Aries, A. (2022). Pengaruh Model Pembelajaran Problem Based Learning (PBL) Terhadap Hasil Belajar Matematika Pada Siswa Kelas VII Di SMP Sultan Agung Surabaya. *Postulat : Jurnal Inovasi Pendidikan Matematika*, 3(1), 1. <https://doi.org/10.30587/postulat.v3i1.4312>

- Callingham, R., & Siemon, D. (2021). Connecting multiplicative thinking and mathematical reasoning in the middle years. *Journal of Mathematical Behavior*, 61(December 2020), 100837. <https://doi.org/10.1016/j.jmathb.2020.100837>
- Carlson, M. P., & Thompson, P. W. (2017). Variation , covariation , and functions : Foundational ways of thinking mathematically. In *Compendium for research in mathematics education* (pp. 421–456) (Issue January).
- Degrande, T., Verschaffel, L., & Van Dooren, W. (2017). Spontaneous Focusing on Quantitative Relations: Towards a Characterization. *Mathematical Thinking and Learning*, 19(4), 260–275. <https://doi.org/10.1080/10986065.2017.1365223>
- Eriksson, H., & Sumpter, L. (2021). Algebraic and fractional thinking in collective mathematical reasoning. *Educational Studies in Mathematics*, 473–491. <https://doi.org/10.1007/s10649-021-10044-1>
- Fuentealba, C., Sánchez-Matamoros, G., Badillo, E., & Trigueros, M. (2017). Thematization of derivative schema in university students: nuances in constructing relations between a function's successive derivatives. *International Journal of Mathematical Education in Science and Technology*, 48(3), 374–392. <https://doi.org/10.1080/0020739X.2016.1248508>
- Habsyi, R., R. M. Saleh, R., & Isman M. Nur. (2022). Pengembangan E-LKPD Berbasis Guided Discovery Learning untuk Meningkatkan Kemampuan Berpikir Kritis Siswa. *Kognitif: Jurnal Riset HOTS Pendidikan Matematika*, 2(1), 1–18. <https://doi.org/10.51574/kognitif.v2i1.385>
- Hackenberg, A. J., Aydeniz, F., & Jones, R. (2021). Middle school students ' construction of quantitative unknowns *. *Journal of Mathematical Behavior*, 61(December 2020), 100832. <https://doi.org/10.1016/j.jmathb.2020.100832>
- Hollebrands, K. F., & Lee, H. S. (2020). Effective design of massive open online courses for mathematics teachers to support their professional learning. *ZDM - Mathematics Education*, 52(5), 859–875. <https://doi.org/10.1007/s11858-020-01142-0>
- Jones, S. R. (2017). An exploratory study on student understandings of derivatives in real-world, non-kinematics contexts. *Journal of Mathematical Behavior*, 45, 95–110. <https://doi.org/10.1016/j.jmathb.2016.11.002>
- Jones, S. R., & Kuster, G. E. (2021). Examining students ' variational reasoning in differential equations. *Journal of Mathematical Behavior*, 64(January), 100899. <https://doi.org/10.1016/j.jmathb.2021.100899>
- Kim, Y. R., Park, M. S., Moore, T. J., & Varma, S. (2013). Multiple levels of metacognition and their elicitation through complex problem-solving tasks. *Journal of Mathematical Behavior*, 32(3), 377–396. <https://doi.org/10.1016/j.jmathb.2013.04.002>
- Kotto, M. A., Babys, U., & Gella, N. J. M. (2022). Meningkatkan Kemampuan Penalaran Matematika Siswa Melalui Model PBL (Problem Based Learning). *Jurnal Sains Dan Edukasi Sains*, 5(1), 24–27. <https://doi.org/10.24246/juses.v5i1p24-27>
- Lee, Y., Capraro, R. M., & Bicer, A. (2019). Affective Mathematics Engagement: a Comparison of STEM PBL Versus Non-STEM PBL Instruction. *Canadian Journal of Science, Mathematics and Technology Education*, 19(3), 270–289. <https://doi.org/10.1007/s42330-019-00050-0>
- Makitalentu, C., Manurung, O., & Regar, V. E. (2023). Penerapan Model Problem Based Learning Pada Pembelajaran Relasi Dan Fungsi. *Jurnal Sosial Humaniora Sigli*, 6(2), 767–770.
- Mohamed Abdul-Rahmana, M. A. M. (2020). Effectiveness of learner control and program control strategies in developing mathematical thinking for slow learners in mathematics. *International Journal of Innovation, Creativity and Change*, 12(12).

- Nolaputra, A. P., Wardono, & Supriyono. (2018). Analisis Kemampuan Literasi Matematika pada Pembelajaran PBL Pendekatan RME Berbantuan Schoology Siswa SMP. *Prisma, Prosiding Seminar Nasional Matematika*, 1.
- Nur, M. A. (2024). Meta Analisis Pengaruh Model Pembelajaran Contextual Teaching and Learning (CTL) terhadap Hasil Belajar Matematika Siswa Sekolah Dasar. *Kognitif: Jurnal Riset HOTS Pendidikan Matematika*, 4(May), 151–160. <https://doi.org/https://doi.org/10.51574/kognitif.v4i1.1409>
- OECD. (2018). *Programme for International Students Assesment (PISA) Result From PISA 2018*.
- Ostermann, A., Leuders, T., & Nückles, M. (2018). Improving the judgment of task difficulties: prospective teachers' diagnostic competence in the area of functions and graphs. *Journal of Mathematics Teacher Education*, 21(6), 579–605. <https://doi.org/10.1007/s10857-017-9369-z>
- Putranto, S., & Marsigit, M. (2018). Is it Effective using Peer Tutoring with Realistic Mathematics Education Approach to Improve Slow Learners' Mathematics Attitudes? *International Journal on Emerging Mathematics Education*, 2(2). <https://doi.org/10.12928/ijeme.v2i2.10487>
- Rahayuningsih, S., Nurasarawati, N., & Nurhusain, M. (2022). Komparasi Efektivitas Model Pembelajaran Project Based Learning (PjBL) dan Konvensional: Studi Pada Siswa Menengah Pertama. *Kognitif: Jurnal Riset HOTS Pendidikan Matematika*, 2(2), 118–129.
- Son, J. W., & Lee, M. Y. (2021). Exploring the Relationship Between Preservice Teachers' Conceptions of Problem Solving and Their Problem-Solving Performances. *International Journal of Science and Mathematics Education*, 19(1), 129–150. <https://doi.org/10.1007/s10763-019-10045-w>
- Tallman, M. A., & Frank, K. M. (2020). Angle measure, quantitative reasoning, and instructional coherence: an examination of the role of mathematical ways of thinking as a component of teachers' knowledge base. *Journal of Mathematics Teacher Education*, 23(1), 69–95. <https://doi.org/10.1007/s10857-018-9409-3>
- Tran, T., Nguyen, T. T. T., Le, T. T. T., & Phan, T. A. (2020). Slow learners in mathematics classes: the experience of Vietnamese primary education. *Education 3-13*, 48(5). <https://doi.org/10.1080/03004279.2019.1633375>
- Viirman, O. (2015). Explanation , motivation and question posing routines in university mathematics teachers ' pedagogical discourse : a commognitive analysis. *International Journal of Education in Science and Techology*, 48(8), 1165–1181. <https://doi.org/10.1080/0020739X.2015.1034206>
- Wilkie, K. J. (2020). Investigating Students' Attention to Covariation Features of their Constructed Graphs in a Figural Pattern Generalisation Context. *International Journal of Science and Mathematics Education*, 18(2), 315–336. <https://doi.org/10.1007/s10763-019-09955-6>

Author Biographies

	Agnes Pesta Kristiani Pardede was born in Medan, Indonesia, on May 9, 2002. She is the first of two children, born to Mr. Remanjo Pardede and Mrs. Veni Manna Sentosa Br Simare-mare, S.Pd. She completed her primary education at SD Tunas Wana Harapan, Riau (2008–2014), and continued her secondary education at SMP Tunas Wana Harapan (2014–2017). She then attended SMK Swasta Raksana Medan (2017–2020). Through the <i>SNMPTN</i> admission program, she was accepted into the Department of Mathematics, Bachelor of Mathematics Education Program, Faculty of Mathematics, Natural Sciences, and Earth Sciences (FMIPAK), Manado State University (Universitas Negeri Manado/UNIMA) in 2020. Her academic interests focus on mathematics education and innovative learning models. Email: agnespestakristiani00@gmail.com
	Philotheus E.A. Tuerah was born in Tomohon, Indonesia. He is a senior lecturer at the Department of Mathematics, Faculty of Mathematics, Natural Sciences, and Earth Sciences (FMIPAK), Manado State University (UNIMA). He earned his Bachelor's degree in Mathematics Education from IKIP Manado (1983), and his Master's degree in Mathematics from Gadjah Mada University, Yogyakarta (1997). He pursued further postgraduate studies in Applied Mathematics at the University of Paris XIII St. Denis, France (1999), and obtained additional Master's degrees in Mathematical Statistics and Bibliometrics from Paul Cézanne University Aix-Marseille III, France (2000). He was awarded a Doctorate (Ph.D.) in Mathematical Statistics and Bibliometrics from the same university in 2002. Email: pheatuerah@unima.ac.id
	Ermita was born in Bone, South Sulawesi, Indonesia. She is a lecturer at the Department of Mathematics, Faculty of Mathematics, Natural Sciences, and Earth Sciences (FMIPAK), Manado State University (UNIMA). She earned her Bachelor's degree in Mathematics Education from Makassar State University (Universitas Negeri Makassar, 2007–2011) and her Master's degree in Mathematics Education through a joint program between the State University of Surabaya (Universitas Negeri Surabaya), Indonesia, and Utrecht University, the Netherlands (2013–2015). Her academic interests include mathematics education, teacher training, and international collaboration in mathematics learning innovation. Email: ermita@unima.ac.id
	Galih Albarra Shidiq, is a Postdoctoral Researcher in the Department of Educational Technology and Communications, Faculty of Education, Chulalongkorn University, Thailand, and also a Lecturer in Indonesia. He holds a PhD from Kasetsart University, Thailand, in 2022. His research interests focus on educational technology, STEM education, science education, and assessment and evaluation. Email: galihalbarra.s@chula.ac.th