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Students' Word-Problem Errors Identified Through Newman's Procedure in a Problem-Based Learning Classroom

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ABSTRACT

Low mathematics achievement, particularly in solving word problems, remains a persistent challenge for students. This study aims to analyze (1) whether students' errors in solving word problems through the Problem-Based Learning (PBL) model are lower than those in conventional learning, (2) whether students' mastery learning in the PBL model is higher than that in conventional learning, and (3) how well students are able to solve mathematical word problems based on Newman's procedure when taught using the PBL model. A mixed methods approach was employed, combining a quasi-experimental design with a qualitative descriptive method. Data were collected through written tests for the quantitative component and in-depth interviews for the qualitative component. The quantitative data were analyzed using an Independent Sample t-test. The findings indicate that students taught with the PBL model experienced a notable reduction in errors across all stages of Newman's procedure. These results demonstrate that the PBL model effectively minimizes student errors, increases mastery learning, and strengthens students' ability to solve mathematical word problems. Overall, the study recommends the use of the PBL model in mathematics classrooms to foster deeper understanding and enhance the relevance of learning to students' everyday experiences.

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Introduction

Mathematics is a fundamental discipline that supports the growth of various fields of science and technology (Swidan, 2020; Swidan & Fried, 2021). Mastery of mathematics is essential not only for academic success but also for developing logical thinking and analytical skills that are useful in everyday life (Scheiner & Pinto, 2019; Sevinc & Lesh, 2022). However, learning mathematics remains a challenge for many students, especially when they are required to solve mathematical word problems that demand comprehension, representation, reasoning, and correct procedural execution (Ikram et al., 2020; Lee et al., 2023; Tallman & Frank, 2020). These tasks often require students to integrate multiple mathematical ideas at once, making word problems a critical yet difficult component of mathematics learning. Consequently, identifying effective teaching approaches that strengthen students' ability to solve word problems has become increasingly important.

International assessments consistently show that students' performance in solving mathematical word problems is still weak. Global studies such as PISA report that many students struggle to interpret information, reason mathematically, and apply concepts in meaningful real-life contexts (OECD, 2018). Similar patterns are observed in Indonesia, where students often demonstrate satisfactory computational skills but experience significant barriers when translating verbal statements into mathematical equations or justifying their solutions (Nurwita et al., 2022; Rich et al., 2019; Wilkie & Hopkins, 2024). These trends indicate a gap between procedural competence and deep mathematical understanding, which reduces students' readiness to tackle higher-order cognitive tasks. Addressing these problems is urgent because difficulties in solving word problems affect students' learning achievement and limit their capacity for real-world decision-making. Therefore, innovative approaches are needed to cultivate students' mathematical literacy and problem-solving ability.

One approach that holds promise is Problem-Based Learning (PBL). PBL encourages students to investigate real problems, discuss solution strategies, and construct their own understanding through active exploration (Y. Lee et al., 2019). Prior research suggests that PBL can foster students' critical thinking and help them build stronger connections between abstract mathematical concepts and practical applications (Kotto et al., 2022; Nolaputra et al., 2018). Scholars also argue that traditional instructional methods, which rely heavily on lecturing, often fail to engage students and may reduce their motivation to engage deeply with mathematical tasks. By integrating real-world contexts and collaborative reasoning, PBL provides opportunities for students to explore problems more meaningfully and to refine their conceptual understanding through discussion and reflection. Consequently, PBL may be particularly effective for reducing student errors in solving word problems.

Preliminary observations reinforce these concerns, showing that many eighth-grade students have difficulty understanding mathematical word problems, particularly topics such as Single-Variable Linear Equations. Students frequently struggle to interpret the situation presented in the problem, identify important information, and convert those ideas into appropriate mathematical forms. These difficulties often lead to errors in constructing equations, selecting strategies, and completing computational steps. Such findings highlight that word-problem errors are not simply computational but are rooted in deeper issues of comprehension and representation. Thus, a more targeted instructional model is needed to help students overcome these specific types of errors.

To analyze these errors more systematically, the Newman Error Analysis (NEA) framework offers a powerful diagnostic tool (Blanton et al., 2015, 2017) NEA classifies student mistakes into five categories: reading errors, comprehension errors, transformation errors, process skill errors, and encoding errors (Paichi Pat Shein, 2012; Tan Sisman & Aksu, 2016). This framework enables teachers and researchers to identify precisely which cognitive stages pose difficulties for students when solving word problems. By understanding these error patterns, teachers can design interventions that address the root causes rather than just the surface-level symptoms. NEA also aligns well with PBL because both emphasize understanding the problem structure and reasoning logically through solution steps. Therefore, integrating PBL with NEA may provide a more comprehensive approach to improving students' problem-solving skills.

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Based on this background, this study aims to analyze the effectiveness of implementing the Problem-Based Learning (PBL) model in improving students' ability to solve mathematical word problems. Specifically, this study investigates whether PBL reduces student errors based on Newman's Procedure, improves students' learning completeness (mastery), and yields better outcomes than conventional learning. The implications of this research are expected to contribute to instructional practices by offering evidence-based strategies for strengthening students' accuracy, comprehension, and reasoning when solving mathematical word problems.

Method

Type of Research

This study employed a mixed-methods approach, integrating a quasi-experimental design and a descriptive qualitative approach. The quantitative approach was conducted through pre-tests and post-tests given to the experimental and control groups, while the qualitative approach utilized in-depth interviews to understand the types and causes of student errors in solving mathematical word problems based on Newman's Procedure. The quasi-experimental design in this study used *pre-tests* and *post-tests* on two groups that were selected non-randomly. The experimental group received treatment with the *Problem-Based Learning* (PBL) model, while the control group received conventional instruction. A comparison of the pre-test and post-test results from both groups was used to analyze the effect of PBL implementation on reducing student errors and increasing *student learning completeness* (mastery). The design of this study is illustrated in Table 1

Table 1. Task Description

Group	*Pre-test*	Treatment	*Post-test*
Experiment	O_1	X	O_2
Control	O_1	-	O_2

Description:

O_1 : *Pre-test* results before treatment
X : *Treatment (*intervention*)* using the *Problem-Based Learning* (PBL) model
\- : Conventional learning methods
O_2 : *Post-test* results after *treatment*

Population and Sample

The population of this study consisted of all eighth-grade students at IT Al Fajar Private Junior High School during the odd semester of the 2024/2025 academic year. This population comprised two classes (VIII-1 and VIII-2), each with 23 students, for a total of 46 students. The study employed a total sampling technique, using the entire population as the sample. Class VIII-1 was designated as the experimental group, receiving the *Problem-Based Learning* (PBL) model, while Class VIII-2 served as the control group, receiving conventional methods.

Instruments

The research instruments consisted of written tests and interviews. The written tests, in the form of pre-tests and post-tests, were designed to identify students' errors in solving mathematical word problems on the topic of Single-Variable Linear Equations within the

Problem-Based Learning (PBL) model. The questions, presented as descriptive word problems, were compiled based on a validated grid, with error analysis referring to *Newman's Error Analysis* (NEA). This analysis encompassed: reading questions, comprehending questions, problem transformation, solving equations, and writing the final answer. The *pre-test* was administered before the treatment to determine the students' initial abilities, while the *post-test* was given after the learning intervention to evaluate learning outcomes.

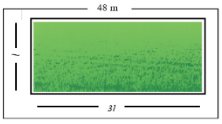
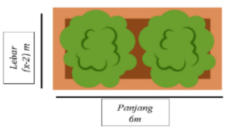
Students' errors in solving mathematical word problems were assessed based on error indicators. The scoring criteria for the student error indicators on the *pre-test* and *post-test* can be seen in Table 2

Table 2. Student Error Indicators According to Newman's Stages

Aspect Assessed	Score	Description
Reading Errors	0	Not filled
	1	Interprets every word, term, or symbol in the question and accurately identifies the variables used.
Comprehension Errors	0	Not filled
	1	Indicates and writes down the known elements without writing down the elements being asked.
	2	Indicates and writes down both the known elements and the elements being asked accurately.
Transformation Errors	0	Not filled
	1	Makes an assumption (for variables) but fails to create a mathematical model.
	2	Makes an assumption (for variables) and accurately creates the mathematical model.
Process Skil Errors	0	Not filled
	1	Interprets every word, term, or symbol in the question and accurately identifies the variables used.
	2	Not filled
	3	Indicates and writes down the known elements without writing down the elements being asked.
Encording Errors	0	Indicates and writes down both the known elements and the elements being asked accurately.
	1	Not filled
	2	Makes an assumption (for variables) but fails to create a mathematical model.

The pre-test and post-test questions used in the study are presented in **Table 3**

Table 3. *Pre-test* and *Post-test* Questions on Student Errors

<table>
<tr><th>Test Type</th><th colspan="3">Question</th></tr>
<tr><td>Pre-test</td><td>1.</td><td>
</td><td>Mr. Hasan has a rectangular plot of land. The length of the land is 3 times its width, and its perimeter is 48 meters. Calculate the area of Mr. Hasan's land!</td></tr>
<tr><td></td><td>2.</td><td>
</td><td>Every school holiday, Aisyah exercises by running 3x km, then continues by walking x km. If the total distance covered by Aisyah is 7, how far does Aisyah run?</td></tr>
<tr><td></td><td>3</td><td>
</td><td>Mr. Hamid has a rectangular garden. The length of the garden is 6 m, while the width is $(x-2)$ m. If the area of Mr. Hamid's garden is 24 m^2, determine the value of x!</td></tr>
</table>

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Post-test	1.		A father was 20 years old when his child was born. How old is the child when their combined ages total 48 years?
	2.		Nani went to the market to buy apples and rambutan. The price of 1 kg of apples is 3 times the price of 1 kg of rambutan at the Murah Meriah store. Nani bought 2 kg of apples and 3 kg of rambutan for Rp90,000.00. If Noni also bought 6 kg of rambutan at the same store, would Rp50,000.00 be enough?
	3.		The price of a chicken is Rp25,000.00 and the price of a goat is Rp650,000.00. Mr. Ridwan wants to buy two goats by selling his chickens. How many chickens does Mr. Ridwan have to sell?

Following the analysis of the students' test results, interviews were conducted with four selected students to obtain more in-depth information regarding the errors identified. These interviews aimed to further identify the causes of errors made by students in solving *Single-Variable Linear Equations word problems*, and to understand the factors that influenced their thought process during problem-solving. The data gathered from these interviews were used to clarify the errors and pinpoint the root causes of the students' difficulties.

Before the test was administered, the instrument had to be validated first. According to Sugiyono (2015), a valid instrument is a measurement tool that is legitimate and appropriate for measuring what it is intended to measure. To assess the validity of the test, evaluation from validators was required. The developed test was validated by two mathematics lecturers and one mathematics teacher from IT Al Fajar Private Junior High School, who assessed the sentence structure, the meaning of the questions, and the accuracy of the answers.

Furthermore, the instrument was also tested for reliability to ensure the consistency of the results obtained. Reliability testing was conducted by piloting the instrument on a sample of students before it was used in the main study. The reliability test results indicated that the test instrument had an acceptable level of consistency and adequate reliability.

The validity of the test items was calculated using the *Product Moment Correlation Coefficient.* An item was considered valid if the value of $r_{count} > r_{table}$. The following are the results of the validity calculations for the *pre-test* and *post-test* items:

Table 4. Pre-Test Item Validation Analysis

Item Number	r_{count}	r_{table}	Description
1	0,808442	0,413	Valid
2	0,767551	0,413	Valid
3	0,734038	0,413	Valid

Table 5. Post-Test Item Validation Analysis

Item Number	r_{count}	r_{table}	Description
1	0,945587	0,413	Valid
2	0,892875	0,413	Valid
3	0,877131	0,413	Valid

Table 4 and Table 5 demonstrate that all test items are valid because the value of $r_{count} > r_{table}$ for every item.

For reliability testing, the *Cronbach's Alpha* technique was employed. An instrument is considered reliable and fit for use in the research if the calculated *r* value is r_{count} > 0,60. maka instrumen dianggap reliabel dan dapat digunakan dalam penelitian ini. Conversely, if the *Cronbach's Alpha* α < 0,60, the test item is considered unreliable .

Table 6. Reliability Criteria

Value of α	Criteria
$\alpha \geq 0{,}80$	Very High
$0{,}60 \leq \alpha < 0{,}80$	High
$0{,}40 \leq \alpha < 0{,}60$	Moderate
$0{,}20 \leq \alpha < 0{,}40$	Low
$\alpha < 0{,}20$	Very Low

The results of the trial test for students' ability to solve mathematical word problems are presented in Table 7.

Table 7. Instrument Reliability Analysis

Item	*Cronbach's Alpha* (α)	Description
Pre-Test	0,649321	Reliabel
Post-Test	0,884886	Reliabel

The reliability test results for students' ability to solve mathematical word problems show that the *Cronbach's Alpha* (α) for the Pre-Test is 0,649321, and for the Post-Test is 0,884886. This indicates that the instruments are reliable (α > 0,60).

Data Collection

In this study, the data collection methods comprised written tests and interviews to obtain relevant data for identifying student errors in solving mathematical word problems on the topic of *Single-Variable Linear Equations* within the *Problem-Based Learning* (PBL) model. The written tests consisted of pre-tests and post-tests, comprising three validated word problems each, which were validated by the supervisor and expert validators. The *pre-test* was administered before the treatment to determine the students' initial abilities, while the *post-test* was given after the learning process with the PBL model to evaluate student learning outcomes and measure the reduction in the error rate during problem-solving. This test was analyzed using *Newman's Error Analysis* (NEA) procedure, which covers five stages of errors: *Reading Errors*, *Comprehension Errors*, *Transformation Errors*, *Process Skill Errors*, and *Encoding Errors*.

Furthermore, interviews were conducted as a supplementary method to deepen the analysis of the written test results. The interviews were free and unstructured, focusing on exploring student errors. They were conducted with four selected students based on their test results: the student with the highest score and the student with the lowest score from each experimental and control class. The purpose of these interviews was to gain a deeper understanding of the students' thought processes during problem-solving, the causes of errors at each Newman stage, and the factors influencing student difficulties. The data from these interviews were used to complement the analysis of the written tests and provide a more comprehensive picture of students' understanding of the material, as well as to assess the contribution of *problem-based learning* (PBL) in helping students reduce errors when solving mathematical word problems.

Data Analysis

Data analysis in this study consisted of quantitative data from the written test results (*pre-test* and *post-test*) and qualitative data obtained through interviews. The quantitative data were analyzed using manual calculations, aided by Microsoft Excel, and statistical testing conducted

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using *SPSS software version 25 for Windows* to ensure result accuracy. The qualitative data from the interviews were analyzed descriptively to deepen the understanding of student errors based on *Newman's Error Analysis* (NEA) procedure. This combined analysis aimed to provide a comprehensive overview of students' ability to solve mathematical word problems and the factors influencing their thought processes.

Descriptive Statistics

To provide a general overview of student performance, the mean, median, and mode scores for the pre-test and post-test of both groups (PBL and conventional learning) were calculated.
The mean score ($\bar{X}$) was calculated using the formula:

$$\bar{X} = \frac{\sum X_i}{N}$$

Where:
$\bar{X}$: Mean (average) score
$\sum X_i$: Sum of all data scores
N : Total number of students

The errors made by students were analyzed based on Newman's Procedure: R*eading Errors*, *Comprehension Errors*, *Transformation Errors*, *Process Skill Errors*, and *Encoding Errors*. The frequency of errors at each stage was calculated and compared between the results in the PBL class and the conventional learning class.

After examination, the test results were analyzed and expressed as a percentage for each test item using the following formula:

$$P_i = \frac{S_i}{z} \times 100\%$$

Where:
P_i: Percentage of errors made by students at the i-th error location
S_i: Number of students who made the error at the i-th error location
z : Total possible errors (Total students × Total items)
i : The position of the error for each test item

Learning Mastery Test

To determine the percentage of learning mastery (completeness), the number of students who achieved the *Minimum Completeness Criteria* (KKM) was calculated in each group.

The formula for the Percentage of Mastery is:

$$P = \frac{R}{N} \times 100\%$$

Where:
P : Percentage of learning mastery (completeness)
R : Number of students who achieved the KKM
N : Total number of students in the class

At IT Al Fajar Private Junior High School, the *Minimum Completeness Criteria* (KKM) for the mathematics subject is set at 75. Based on this value, the students' mastery categories were grouped as shown in Table 8

Table 8. Learning Mastery Categories

Student Score Criteria	Category
Students who score ≥ 75	Complete (Mastery)
Students who score < 75	Not Complete (Not Mastery)

Inferential Statistics Test

Inferential statistical testing was conducted to compare the pre-test and post-test results between the PBL class and the conventional learning class, aimed at identifying significant differences in the improvement of mathematics learning outcomes. The t-test (*Student's t-test*) was employed to examine the difference in the mean scores between the two groups following the implementation of different instructional methods. Before the t-test was carried out, preliminary requirement tests, namely normality and homogeneity tests, were performed.

Results

Results

Analysis of Student Errors Based on Newman's Error Types

Based on Table 9 and the graph above, it can be seen that the implementation of the PBL learning model shows a significant difference compared to the conventional learning method. In the PBL class, no errors were found in reading or comprehending information from the problem, while in the conventional learning class, there were 2 errors (2,90%) and 12 errors (17,39%) for the reading and comprehension stages, respectively. In the transformation stage, 2 errors (2,90%) occurred in the PBL class, where students had difficulty converting the word problem into a mathematical equation. Conversely, 9 errors (13,04%) were recorded in the conventional learning class at the same stage. For the process skill stage, the error frequency in the PBL class was 13 errors (18,84%), which is lower than the conventional learning class, which reached 23 errors (33,33%). Errors in the final answer encoding stage also show a difference. In the PBL class, 26 errors (37,68%) were recorded where students failed to write the final answer, while in the conventional learning class, 32 errors (46,38%) were found at this stage.

Table 9. Frequency and Percentage of Errors Based on Newman's Stages in the PBL and Conventional Learning Groups

Error Stage	Error Location	Error Frequency (PBL)	Error Percentage (PBL)	Error Frequency (Conventional Learning)	Error Percentage (Conventional Learning)
Reading Errors	Students do not correctly understand the problem	0	0%	2	2,90%
Comprehension Errors	Students do not write down the important information in the problem (known and asked)	0	0%	12	17,39%

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Transformation Errors	Students cannot convert the word problem into a mathematical equation	2	2,90%	9	13,04%
Process Skill Errors	Students make errors in calculation	13	18,84%	23	33,33%
Encoding Errors	Students do not write the final answer	26	37,68%	32	46,38%

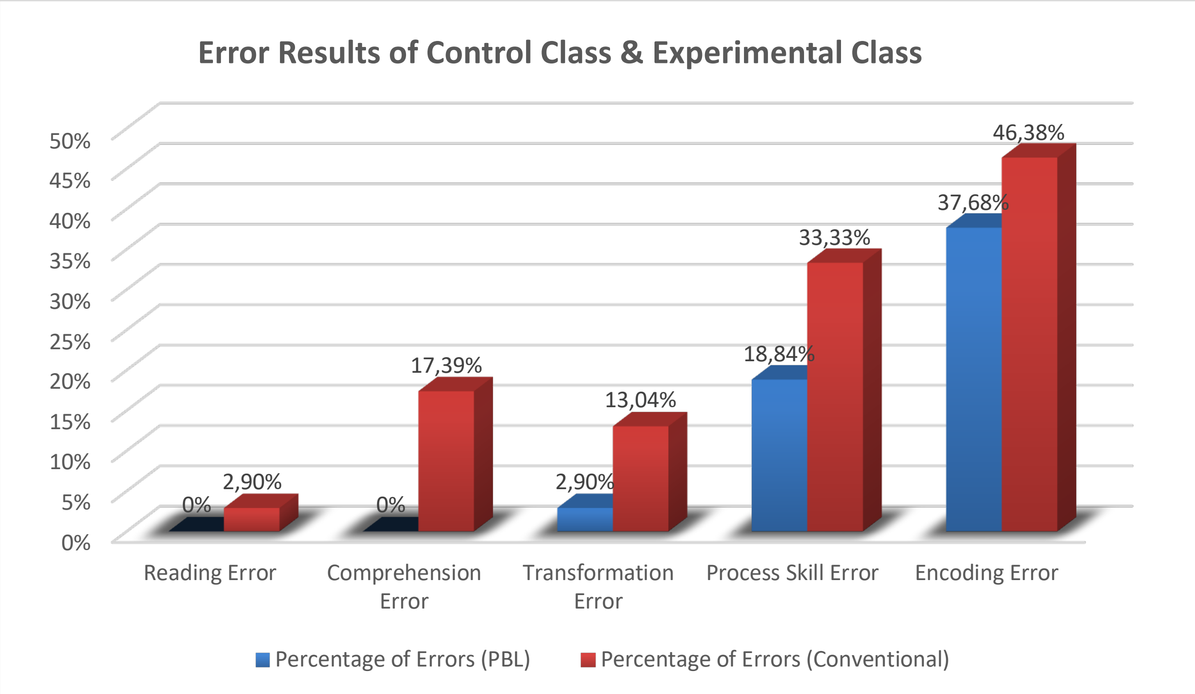

Figure 1. Graph of Errors Percentage Based on Newman's Stages in the PBL and Conventional Learning Groups

Analysis of Student Mastery in Classes VIII-1 and VIII-2

Data on student learning mastery (completeness) in Class VIII-1 (experimental class) and Class VIII-2 (control class) are shown in Table 10

Table 10. Student Mastery Results for Classes VIII-1 and VIII-2

Class	Total Students	Not Complete (Not Mastery)	Complete (Mastery)	Percentage of Mastery
VIII-1 (Experiment)	23	2	21	91,30%
VIII-2 (Control)	23	13	10	43,48%

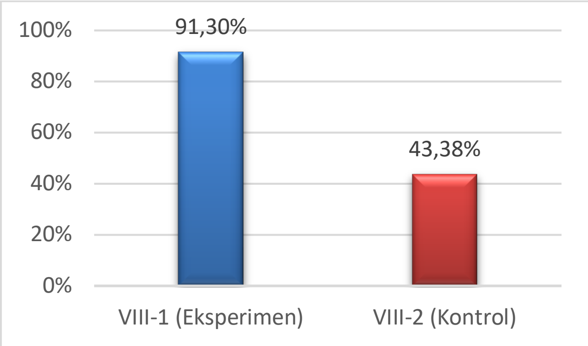

Figure 2. Graph of Student Mastery Percentage

Based on Table 10 and the graph above, student mastery was measured based on the Minimum Completeness Criteria (KKM), where a score of < 75 indicates a student is not yet complete. The percentage of student mastery in each class was obtained by dividing the number of students who achieved mastery by the total number of students and multiplying by 100%. The calculation results show that in the experimental class (VIII-1), out of 23 students, 2 students did not achieve mastery, resulting in 21 students achieving mastery with a mastery percentage of 91,30%. In contrast, in the control class (VIII-2), out of 23 students, 13 students did not achieve mastery, leaving 10 students who achieved mastery with a mastery percentage of 43,48%. This difference indicates that the instructional method implemented in the experimental class, the *Problem-Based Learning* (PBL) model, is superior in increasing student mastery compared to the method used in the control class, which was conventional learning.

Analysis of Data Normality, Homogeneity, and *t*-test

Inferential analysis (normality, homogeneity, and *t*-test) was used to address the research question, specifically whether the student errors in solving word problems through *Problem-Based Learning* (PBL) were not better than those students who received conventional learning at IT Al Fajar Private Junior High School. The data used for this analysis were the post-test results, as the post-test provides an overview of student learning outcomes after the treatment was applied. Data analysis was performed with the aid of the *SPSS program version 25 for Windows.*

Normality Test

The normality test was performed using the Shapiro-Wilk test. The test criterion is that if the significance value (Sig.) is greater than 0,05 (Sig. > 0,05), the data are normally distributed. Conversely, if the significance value is less than 0,05 (Sig. < 0,05), the data are not normally distributed. The normality test results for the post-test data of the experimental class (VIII-1) and the control class (VIII-2) are presented in Table 11

Table 11. Normality Test Results

Class	Sig. Value	Data Distribution
VIII-1 (Experiment)	0,552	Normal
VIII-2 (Control)	0,192	Normal

Homogeneity Test

After the normality test was performed and the data were found to be normally distributed, the next step was the homogeneity test to determine whether the variances between the two data

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groups were homogeneous or not. This test was conducted using *Levene's Test*. The test criterion shows that if the significance value is greater than 0,05 (Sig. > 0,05), the data have homogeneous variance; conversely, if it is less than 0,05 (Sig. < 0,05), the data are not homogeneous. Table 12 below shows the homogeneity test results for the student post-test data:

Table 12. Homogeneity Test Results for Student Post-Test Data

Variable	Sig. Value	Conclusion
Post-test Results	0,171	Homogeneous

t-test (Independent Samples *t*-test)

Since the data met the assumptions of normality and homogeneity, the analysis proceeded with the *t*-test to examine the difference in the mean error scores between the experimental and control classes. The *t*-test was performed using the Independent Samples *t*-test. The test criterion is that if the significance value (Sig. 2-tailed) is less than 0,05, there is a significant difference between the two groups. The results of the t-test analysis are shown in Table 13

Table 13. Independent Samples *t*-test Results for Post-Test Data

Independent Samples Test						
		t-test for Equality of Means				
		T	Df	Sig. (2-tailed)	Mean Difference	Std. Error Difference
Hasil *Post-Test*	Equal variances assumed	5.035	44	.000	13.000	2.582
	Equal variances not assumed	5.035	43.002	.000	13.000	2.582

Based on Table 13, the significance value (Sig. 2-tailed) of 0,000 indicates that there is a significant difference between the student errors in the experimental class and the control class. This result thus shows that learning using the *Problem-Based Learning* (PBL) model has a *significant impact* on reducing student errors in solving mathematical word problems compared to conventional learning.

Analysis Data from Experimental Class Interview (Problem-Based Learning Model)

Presentation of Interview Results with the Highest-Scoring Student

Based on the student answer sheets from the post-test and considering the students' willingness to be interviewed, the student with the code E19 was selected for qualitative analysis. This student obtained the highest final score in the experimental class, achieving a perfect score of 100.

The following is an excerpt from the interview (Notation: Researcher (R), Student (E19)):

R : *Student, observe the significant difference between your pre-test and post-test results. What accounts for this change?*
E19 : *Yes, Ms., this material had actually been taught before, but I had forgotten and was confused about how to solve the problems.*
R : *It is impressive that you achieved a perfect score of 100. Do you consider these questions to be easy to solve?*
E19 : *They were not excessively easy, Ms.*
R : *Then, how were you able to complete all the problems correctly?*

E19 : *It is because you taught me, Ms., and I also enjoyed the instruction you provided; it was easy to understand. Especially when you showed us video presentations of problems that I frequently encounter in my daily life, I was then able to complete the questions you gave.*

R : *In that case, what is your opinion on learning through a group system, as we did during the lesson?*

E19 : *I believe learning in groups is more engaging, Ms. We can help each other when solving problems. If I did not know an answer, a friend would teach me, and if my friend struggled, I would take turns teaching them. Although perhaps some classmates did not wish to participate in the discussion, I ultimately understood the material much better.*

R : *Excellent. Let me test your understanding again. What is the mathematical model for question number 1?*

E19 : *For question number 1, Ms., the mathematical model is, for example, the child's age is* x, *and the father's age is* $x + 20$. *Therefore, the equation is* $x + (x + 20) = 48$. *This is because the question asks for the child's age when the sum of the father's and child's ages is 48 years, and it is known that the father was 20 years old when the child was born.*

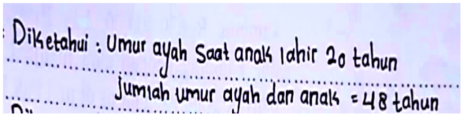
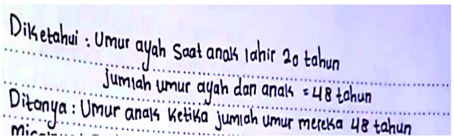
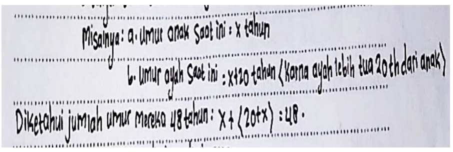
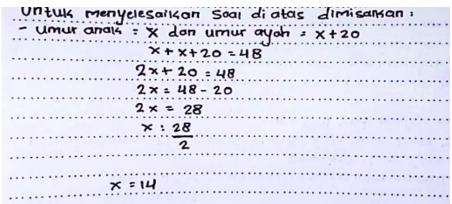
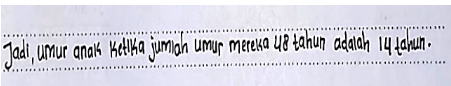
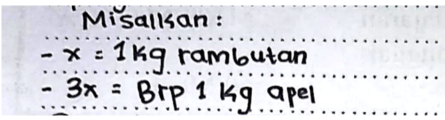
R : *That is correct. In question number 2, why did you conclude that Noni did not have enough money?*

E19 : *Yes, Ms., because I calculated that 1 kg of rambutan costs Rp. 10,000. Noni bought 6 kg of rambutan, which means she must pay Rp. 60,000. However, Noni only brought Rp. 50,000, so her money was insufficient.*

R : *Very good, that is also correct. For question number 3, please explain to me how you solved this problem.*

E19 : *First, I calculated the total cost for the two goats by multiplying the price per goat by two, Ms. Subsequently, I divided that total cost by the price per chicken to determine the number of chickens Pak Ridwan needed to sell.*

Table 14. Final Test Answer Results of Student E19

No.	Newman's Error Stage	Student Answer Result (Placeholder)	Criteria/Description
	Reading Errors		Students can correctly comprehend the information presented.
	Comprehension Errors		Students are able to properly understand the context of the problem.
1	Transformation Errors		Students are able to correctly convert the word problem into an equation.
	Process Skills Errors		Students are able to demonstrate problem-solving skills effectively (i.e., performing calculations).
	Encoding Errors		Students are able to write the final answer clearly and consistent with the solution results.
2	Reading Errors		Students are able to correctly comprehend the information presented.

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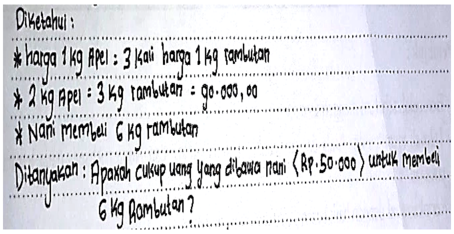
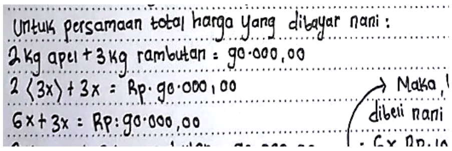
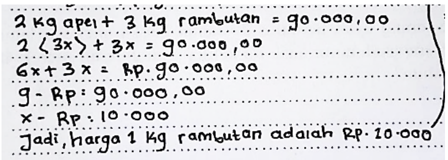
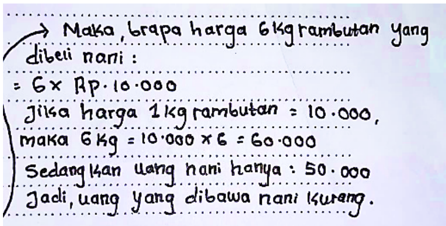
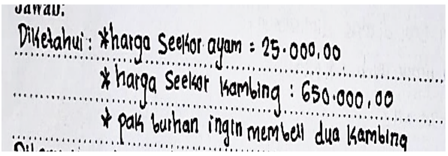
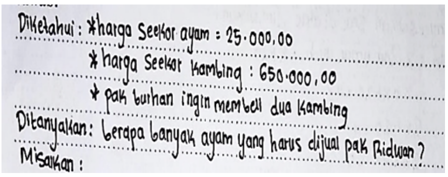
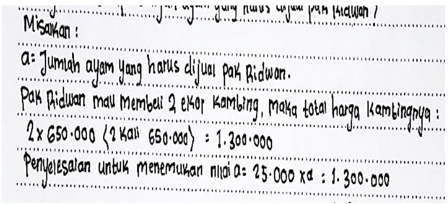
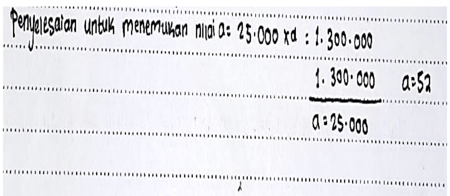
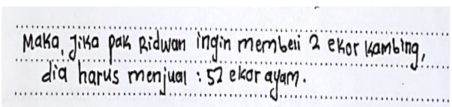
No.	Newman's Error Stage	Student Answer Result (Placeholder)	Criteria/Description
	Comprehension Errors	 Diketahui : * harga 1 kg Apel : 3 kali harga 1 kg rambutan * 2 kg Apel + 3 kg rambutan : 90.000,00 * Nani membeli 6 kg rambutan Ditanyakan : Apakah cukup uang yang dibawa nani (Rp.50.000) untuk membeli 6 kg Rambutan ?	Students are able to properly understand the context of the problem.
	Transformation Errors	 Untuk persamaan total harga yang dibayar nani : 2 kg apel + 3 kg rambutan : 90.000,00 2 (3x) + 3x : Rp. 90.000,00 6x + 3x : Rp: 90.000,00 Maka, dibeli nani	Students are able to correctly convert the word problem into an equation.
	Process Skills Errors	 2 kg apel + 3 kg rambutan = 90.000,00 2 (3x) + 3x = 90.000,00 6x + 3x = Rp. 90.000,00 9 - Rp: 90.000,00 x - Rp: 10.000 Jadi, harga 1 kg rambutan adalah Rp. 10.000	Students are able to demonstrate problem-solving skills effectively.
	Encoding Errors	 Maka, brapa harga 6 kg rambutan yang dibeli nani : = 6x Rp. 10.000 Jika harga 1 kg rambutan = 10.000, maka 6 kg = 10.000 x 6 = 60.000 Sedangkan uang nani hanya : 50.000 Jadi, uang yang dibawa nani kurang.	Students are able to write the final answer clearly and consistent with the solution results.
	Reading Errors	 Diketahui : * harga Seekor ayam : 25.000,00 * harga Seekor kambing : 650.000,00 * pak burhan ingin membeli dua kambing	Students are able to correctly comprehend the information presented.
	Comprehension Errors	 Diketahui : * harga Seekor ayam : 25.000,00 * harga Seekor kambing : 650.000,00 * pak burhan ingin membeli dua kambing Ditanyakan : berapa banyak ayam yang harus dijual pak Ridwan ? Misalkan :	Students are able to properly understand the context of the problem.
3	Transformation Errors	 Misalkan : a: Jumlah ayam yang harus dijual pak Ridwan. Pak Ridwan mau membeli 2 ekor kambing, maka total harga kambingnya : 2x 650.000 (2 kali 650.000) : 1.300.000 Penyelesaian untuk menemukan nilai a: 25.000 xa : 1.300.000	Students are able to correctly convert the word problem into an equation.
	Process Skills Errors	 Penyelesaian untuk menemukan nilai a: 25.000 xa : 1.300.000 1.300.000 / a=25.000 a=52	Students are able to demonstrate problem-solving skills effectively.
	Encoding Errors	 Maka, jika pak Ridwan ingin membeli 2 ekor kambing, dia harus menjual : 52 ekor ayam.	Students are able to write the final answer clearly and consistent with the solution results.

Based on Table 14, the interview transcripts, and the student's work on the presented word problems, it is evident that the student coded E19 made no errors in the solution process. This finding indicates that Student E19 has achieved a strong conceptual understanding of the material. Furthermore, based on this result, it can be concluded that learning using the *Problem-Based* Learning (PBL) model is effective in reducing student errors in solving mathematical word problems and enhancing their comprehension of the concept of linear equations in one variable.

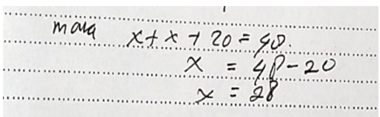
Presentation of Interview Results with the Lowest-Scoring Student

Based on the student answer sheets from the final test and considering the students' willingness to be interviewed, the student with the lowest score in the experimental class, coded E16, was selected for qualitative analysis, having obtained a final score of 67.

The following is an excerpt from the interview (Notation: Researcher (R), Student (E16)):

R : Student, please examine this calculation of yours. Is this correct? Try counting it again.
E16 : Oh, yes, Ms. I apologize, I was less careful in calculating it.
R : So, what should the final equation be?
E16 : It should be $x + x + 20 = 48$, *Ms., so* $2x + 20 = 48$.
R : So, the result was incorrect, wasn't it?
E16 : Yes, Ms., I apologize. I rushed because I was afraid the time would run out.
R : Regarding question number 2, why did you write this conclusion? Did you recheck your answer after finishing? Please read the question again. What exactly is being asked?
E16 : Oh, yes, Ms., the question is: if Noni also bought 6 Kg of rambutan from the same store, would Rp. 50,000 be sufficient?
R : You answered that the price of 1 kg of rambutan is Rp. 10,000. What should be the subsequent step?
E16 : It should be multiplied by 6, Ms., because Noni bought 6 kg of rambutan, so the cost is Rp. 60,000. Thus, if Noni only brought Rp. 50,000, her money would be insufficient.
R : That is correct. Why did you not write that down?
E16 : Yes, Ms., I apologize. I was rushing because I was afraid I wouldn't be able to solve question 3, so I lost focus on what was actually being asked.
R : Next time, you must be meticulous when solving mathematical problems, and read the question repeatedly if necessary to avoid mistakes. Also, why did you not write the conclusion for question number 3?
E16 : Because the allocated time had run out, Ms.
R : In your opinion, student, what do you think of the learning method I provided to you all?
E16 : It was enjoyable, Ms., it was interesting, Ms.
R : Why do you say that? When I divided you into groups, did you find that you understood better when studying with your classmates?
E16 : Yes, Ms., it was exciting when you asked questions while showing a video, Ms. I prefer learning in groups, Ms., but I feel embarrassed to ask my friends if I don't know the answer.
R : Why are you embarrassed?
E16 : Yes, Ms., I am embarrassed because I don't know the answer.
R : Next time, you do not need to feel embarrassed to ask your friends, student, because we are all learning together.

Table 15. Final Test Answer Results of Student E16

No.	Newman's Error Stage	Student Answer Result (Placeholder)	Criteria/Description
1	Proses Skill Errors		The student made an error in the transformation stage, specifically in converting the word problem into the correct equation. This error occurred in the improper application of algebraic operations when attempting to find the value of *x*.

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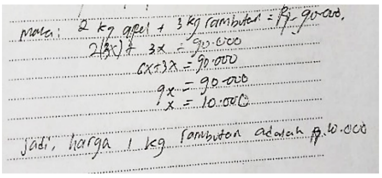
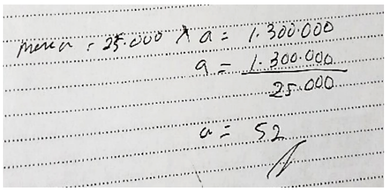
No.	Newman's Error Stage	Student Answer Result (Placeholder)	Criteria/Description
2	Encoding Errors		The student made an error in this stage because they did not continue the calculation to the final step as instructed by the question. The student only calculated the price per kilogram (Rp. 10,000) but failed to multiply it by the corresponding quantity in the problem, which was 6 kg of rambutan, to determine if the amount of money brought was sufficient. Consequently, the provided answer was incomplete and did not address the question asked.
3	Encoding Errors		The student made an error by only writing the final numerical result (52) without drawing a conclusion relevant to the problem. The student failed to link the calculation result to the question asked, making the answer incomplete and lacking a clear explanation of the problem-solving process.

From Table 15, the interview transcript, and the student's work on the given word problems, the errors committed by the student coded E16 involve the Process Skills Errors and Encoding Errors stages. Based on the student's statement, the cause of the Process Skills Errors was the student rushing to solve the problems, which resulted in calculation mistakes. Meanwhile, the cause of the Encoding Errors was attributed to the student's lack of diligence in re-examining the final result and running out of time during the examination, which led to errors in concluding the calculation results. Consequently, it can be concluded that Student E16 experienced errors in the aspects of process skills and encoding the final answer.

Analysis Data from Control Class Interview (Conventional Learning)

Presentation of Interview Results with the Highest-Scoring Student

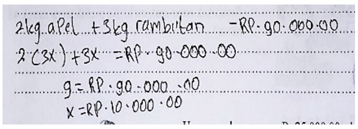
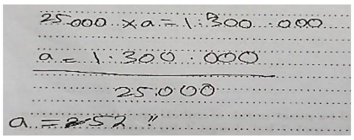
Based on the student answer sheets from the final test and considering the students' willingness to be interviewed, the student with the code K23 was selected for qualitative analysis. This student obtained the highest final score in the control class, achieving a score of 87.

The following is an excerpt from the interview (Notation: Researcher (R), Student (K23)):

R : Student, can you explain or state all the known information from question number 1?
K23 : Yes, Ms. The known information in question number 1 is the father's age when the child was born, which is 20 years, and the sum of the child's and father's ages, which is 48 years.
R : In question number 2, why did you not write the conclusion, student?
K23 : Oh yes, Ms. I forgot, Ms.
R : Please re-read question number 2. What is being asked in the problem?
K23 : If Noni also bought 6 kg of rambutan from the same store, would Rp. 50,000 be sufficient?
R : Alright, please look at your answer again.
K23 : Oh yes, Ms., I made a mistake; I was not focused on what the question was asking.
R : Why were you not focused, student?

K23 : *Because I was also rushing, Ms., afraid that the time would run out and I wouldn't be able to answer question number 3.*
R : *Next time, please be more meticulous when solving mathematical problems. Now, I will ask again: similarly, in question number 3, you did not provide a conclusion in your answer. Why is that, student?*
K23 : *Yes, Ms., it's the same; I rushed through it because I was afraid the time would run out.*

Table 16. Final Test Answer Results of Student K23

No.	Newman's Error Stage	Student Answer Result (Placeholder)	Criteria/Description
2	Encoding Errors		The error occurred because the student failed to match the calculation result with the question asked in the problem, resulting in an inappropriate or missing final conclusion.
3	Encoding Errors		The error occurred because the student failed to explain the relationship between the final numerical result and the context of the question, thus the answer did not provide a clear solution.

From Table 16, the interview transcript, and the student's work on the given word problems, the errors committed by Student K23 involved the Encoding Errors stage, or the final conclusion writing. Based on the student's statement, the cause of the Encoding Errors (in writing the final answer or omitting the conclusion) was the student's lack of diligence and rushing while writing the answer. This resulted in mistakes and forgetting to write down the final outcome of the work. Consequently, it can be concluded that Student K23 primarily experienced errors in the aspect of encoding the final answer.

Presentation of Interview Results with the Lowest-Scoring Student

Based on the student answer sheets from the final test and considering the students' willingness to be interviewed, the student with the lowest score in the control class, coded K9, was selected for qualitative analysis, obtaining a final score of 53.

The following is an excerpt from the interview (Notation: Researcher (R), Student (K9)):

R : *Student, why did you not complete your work on question number 1?*
K9 : *Yes, Ms., I forgot how to solve it.*
R : *Regarding question number 2, why did you not provide a complete answer?*
K9 : *Yes, Ms., I was confused about how to continue the calculation results.*
R : *Now, for question number 3, you answered with the mathematical model* $25000 \times x = 650.000$. *Please re-read this sentence in question number 3.*
K9 : *Mr. Ridwan wants to buy two goats by selling his chickens. How many chickens must Mr. Ridwan sell?*
R : *Correct. Mr. Ridwan wants to buy two goats, so you should multiply the price of the goat by 2, yielding* $2 \times 650.00 = 1.300.000$. *After that, you can form an equation to calculate how many chickens must be sold:* $25.000 \times x = 1.300.000$.
K9 : *I was confused about what to write, Ms., so that is what I wrote.*
R : *Why did you have difficulty answering question number 3?*
K9 : *Yes, Ms., I did not know how to convert it into this type of equation.*

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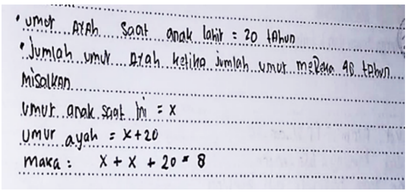
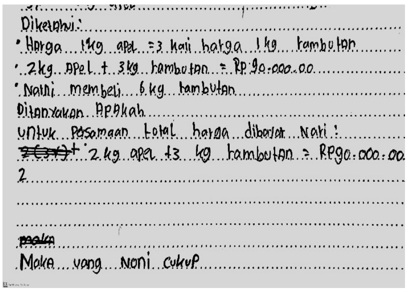
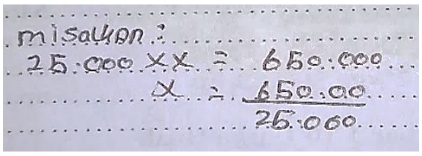
Table 17. Final Test Answer Results of Student K9

No.	Newman's Error Stage	Student Answer Result (Placeholder)	Criteria/Description
1	Process Skills Errors		The student was unable to perform the mathematical operations correctly, resulting in an answer inconsistent with the required solution.
2	Process Skills Errors and Encoding Errors		The student failed to complete the solution process correctly but immediately wrote a conclusion, leading to an inaccurate answer.
3	Transformation Errors		The student made an error in constructing the mathematical model from the word problem, which led to subsequent mistakes in the solution steps and the final result.

From Table 17, the interview transcript, and the student's work on the given word problems, it is evident that Student K9 committed several errors, specifically involving Process Skills Errors, Transformation Errors, and Encoding Errors. Based on the student's statement, the Process Skills Errors occurred because the student did not know how to continue the calculation after the initial step. In the Transformation Errors stage, the student had trouble converting information from the word problem into the correct mathematical equation. Meanwhile, the Encoding Errors (in writing the final answer or conclusion) were caused by a lack of diligence in rechecking the calculation results and a fear of running out of time, which led the student to make mistakes in formulating the conclusion. Overall, it can be concluded that the student coded K9 experienced difficulties in the aspects of process skills, transformation, and encoding the final answer.

Discussion

The findings of this study show that the Problem-Based Learning (PBL) model plays an important role in reducing student errors when solving mathematical word problems, particularly in the transformation and process skills stages (Kotto et al., 2022; Y. Lee et al., 2019; Nolaputra et al., 2018). Students in the PBL class demonstrated stronger abilities to interpret problem contexts, identify relevant information, and translate verbal statements into mathematical equations compared to students who received conventional instruction. This improvement indicates that the structured problem-solving environment provided by PBL helps students deepen their conceptual understanding and apply mathematical procedures more accurately. These results are consistent with Kotto et al. (2022), who reported that PBL enhances students' learning outcomes by encouraging active engagement and collaborative exploration. Although PBL is not without challenges, especially for students with weaker academic backgrounds, it remains a promising approach for improving students' mathematical reasoning and problem-solving abilities.

Interviews and student work analyses revealed five categories of errors based on Newman's Procedure, offering deeper insight into how students engage cognitively with word problems. Reading errors emerged when students failed to extract essential information due to difficulty understanding problem statements, while comprehension errors occurred when students misinterpreted the conceptual meaning of a problem (Björklund & Palmér, 2022). Transformation errors were typically associated with difficulties translating verbal descriptions into mathematical equations, demonstrating a gap between understanding and representation (Chronaki & Planas, 2018; Fonger, 2019; Montenegro et al., 2018; van Lieshout & Xenidou-Dervou, 2018). Some students committed process skills errors by applying incorrect algorithms or performing inaccurate calculations, often due to carelessness or limited procedural fluency. Encoding errors occurred when students, despite reaching the correct process, struggled to express their final answers accurately, showing a lack of attention to concluding steps in problem-solving.

These findings support prior research showing that conceptual and representational weaknesses are major contributors to student errors in mathematical word problems. Studies by Thompson et al. (2014) and Hughes et al. (2020) similarly noted that students often fail to construct accurate mathematical models due to incomplete comprehension of the problem structure. In this study, comprehension and process skill errors were the most prevalent, indicating that students continue to face challenges not only in understanding the language of mathematics but also in executing procedures consistently. The PBL model appears to mitigate some of these issues by providing opportunities for students to articulate their reasoning and negotiate meaning with peers. However, the persistence of certain errors suggests that PBL alone may not fully address shortcomings in students' procedural accuracy.

The comparison between the PBL and conventional learning groups provides further evidence of the model's effectiveness in supporting student mastery. In the PBL group, most students demonstrated learning completeness, whereas fewer than half of the students in the conventional group achieved the same level of success. This difference suggests that PBL provides a more supportive environment for students to build deeper conceptual understanding and confidence in solving word problems. The active learning structure of PBL encourages students to construct knowledge through problem exploration, discussion, and reflection, which aligns with findings by Kotto et al. (2022), who reported improved mastery among students using PBL. These results affirm that instructional approaches grounded in inquiry and real-world problem contexts may be more effective than traditional teacher-centered methods.

Although PBL proved effective in reducing initial reading and comprehension errors, higher rates of process skills and encoding errors remain a concern. This pattern indicates that while students gain a stronger understanding of problem contexts, they require additional support in executing calculations accurately and expressing final answers correctly (Paichi Pat Shein, 2012; Tan Sisman & Aksu, 2016). Errors at these later stages may stem from limited procedural fluency, lack of checking habits, or cognitive overload during multi-step problem solving. This aligns with Lee et al. (2019), who argue that PBL may challenge students who lack strong foundational skills, potentially causing difficulties during the calculation stages. Therefore, supplementary strategies such as structured practice, explicit instruction in mathematical procedures, and guided reflection on solution steps may be necessary to complement PBL.

Overall, the results indicate that PBL provides substantial benefits in improving students' problem-solving abilities and reducing errors in the early stages of word-problem processing. Students in the PBL class demonstrated better comprehension, fewer misinterpretations, and greater confidence in identifying relevant information. Despite persistent errors in some

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procedural and encoding aspects, the general reduction in errors suggests improved cognitive engagement and conceptual understanding. These findings position PBL as a pedagogical approach that not only improves learning outcomes but also enhances students' readiness to solve realistic problems. Nonetheless, to maximize its effectiveness, PBL should be integrated with additional strategies that reinforce accuracy, procedural fluency, and the ability to articulate complete solutions.

Conclusion

The study provides evidence that the Problem-Based Learning (PBL) model fosters deeper engagement with mathematical word problems and significantly reduces student errors across Newman's stages. The effectiveness of PBL is reflected not only in improved comprehension and problem representation but also in students' enhanced ability to sustain coherent reasoning throughout the solution process. These findings affirm theoretical perspectives that position PBL as a catalyst for higher-order thinking, as it places students in authentic problem contexts that require the integration of multiple cognitive processes. While PBL demonstrates strong potential in strengthening conceptual understanding, persistent difficulties at the process and encoding stages indicate the need for complementary instructional supports that reinforce procedural fluency and accuracy. A limitation of this study is that it focuses on a single school context and a specific mathematical topic, which may constrain the generalizability of the findings. Future research should examine the implementation of PBL across broader student populations, varied school settings, and different mathematical domains to further refine its applicability and identify conditions that optimize its effectiveness.

Conflict of Interest

The authors declare that there is no conflict of interest.

Authors' Contributions

H.A understood the research ideas presented, collected data, analyzed the data, discussed the results, and approved the final version of the work. E.S. actively participated in the development of methodological theory and instruments. All authors declare that the final version of this paper has been read and approved. The total percentage contribution for conceptualization, drafting, and revising of this paper is as follows: H.A.: 70%, and E.S.: 30%.

Data Availability Statement

The authors state that the data supporting the findings of this study will be made available by the corresponding author, [H.A.], upon reasonable request.

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