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Design of an RME-Based Fractions Textbook in the Luwu Tourism Context to Stimulate Students' Reasoning Skills

Muhammad Ikram[1], Marufi[2], Suriani Canne[3]
[1, 2, 3] Departement of Mathematics Education, Faculty Teacher Training and Education, Universitas Cokroaminoto Palopo, Indonesia

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ABSTRACT

Students in primary school must think mathematically. However, classroom research shows that students' fraction reasoning is low and teachers' instructional materials don't permit higher-order thinking. This highlights the need for a more relevant and meaningful textbook that matches students' learning patterns. Therefore, this study develops and tests a fractions textbook based on Realistic Mathematics Education (RME) using Luwu tourism to improve primary students' reasoning skills. Two teaching experiment cycles were used for R&D. A total of 23 fifth graders and one teacher from Primary School 5 Tarabbi Indah participated in the study. Expert validation sheets, reasoning rubrics, examinations, and instructor/student questionnaires were used. Research included needs analysis, textbook design, expert validation, first-cycle field testing, product change, second-cycle field testing, and retrospective analysis. Data were descriptively investigated using validation, reasoning, user responses, and cycle comparisons. The findings indicated that the textbook was highly legitimate in content, construct, language, and contextual alignment, according to experts. The first cycle teacher gave 3.8, and the second gave 3.9, while students gave 3.63 and 3.44 in the "practical" category. The textbook worked, as sufficient and very competent reasoning skills increased from 47.8% in the first cycle to 69.6% in the second. Understanding information, evaluating tactics, and presenting mathematical arguments improved. The textbook modifications made these improvements possible by simplifying directions, illustrations, and key subjects. This work improves contextual RME-based teaching materials conceptually and practically. The textbook can assist primary school teachers and schools in planning meaningful, relevant fraction instruction that promotes higher-order thinking.

Corresponding Author:

Muhammad Ikram,
Departement of Mathematics Education, Faculty Teacher Training and Education, Universitas Cokroaminoto Palopo, Indonesia
Email: muhammad.ikram@unm.ac.id

1. INTRODUCTION

A fundamental mathematical competency that must be cultivated in pupils, in accordance with the vision of mathematics education and 21st-century capabilities, is

reasoning (Beswick & Fraser, 2019; Liu et al., 2015). The reasoning category has shown a notable deterioration in Indonesia's performance in the 2018 PISA findings and the Trends in International Mathematics and Science Study (Kanes et al., 2014; OECD, 2018; Safrudiannur & Rott, 2019), necessitating intervention through instructional activities. Reasoning necessitates that individuals concentrate on patterns, structures, or regularities when addressing difficulties (Jäder et al., 2020; Lithner, 2017; Muzaini et al., 2022). Consequently, sound thinking serves as a fundamental reference in educational activities, as it aids individuals in making judicious selections.

The suboptimal reasoning skills of pupils at both international and national levels persistently motivate educators to innovate in enhancing mathematical reasoning in students (Mueller et al., 2014; Smit et al., 2017). This suboptimality is partially attributed to students' lack of acquaintance with real-world problems associated with mathematical ideas (Ikram et al., 2018; Ikram et al., 2020a, 2020b). Students can utilize reasoning skills more efficiently when they are familiar with engaging in real-life challenges (Prahmana et al., 2023). This fact suggests that issues must be aligned with authentic circumstances seen by students (Mata-Pereira & da Ponte, 2017). An effective pedagogical strategy is thus required to encourage pupils to adapt to resolving real-world issues.

Fractions are a subject area evaluated in PISA and TIMSS assessments. Students encounter numerous challenges when addressing fraction problems, including difficulties in interpreting word problems (Tobias, 2013; Wijaya, 2017) and manipulating fractions (Copur-Gencturk, 2021). Our initial research Ikram et al. (2022) revealed that the majority of students encountered difficulties with contextual fraction problems. The primary issue noted was that students' challenges stemmed from the restricted diversity of problems in the textbook, which predominantly featured strictly procedural exercises (for instance, 1/3 + 1/2 = …, 3 2/4 + 2 1/3 = …). This scenario prompts the current study to create a more contextualized and realistic fractions textbook.

A remedy for the deficiency of contextual problems in textbooks is to create a fractions textbook specifically focused on real-world or contextual issues. The Realistic Mathematics Education (RME) approach serves as an appropriate framework for this purpose. RME provides a pedagogical approach that begins with significant real-world contexts and progressively transitions to more formal mathematical principles (Fredriksen, 2021; Hasbi et al., 2019; Prahmana, 2022). This approach is based on human action, primarily aiming to facilitate the transfer from informal to formal knowledge via contextual challenges (Papadakis et al., 2017; Yilmaz, 2020). The RME technique serves as a crucial basis for textbook design intended to enhance students' mathematical reasoning.

The settings of the questions that initiate students' reasoning activities intricately connect to the RME technique. Prior research has extensively examined RME within cultural contexts (Muslimin et al., 2020; Nursyahidah et al., 2021; Pathuddin et al., 2021a; Prahmana, 2017; Prahmana & D'Ambrosio, 2020; Risdiyanti & Prahmana, 2021). These studies demonstrate how local cultural practices can serve as valuable

foundations for mathematization. To our knowledge, no research has utilized RME within a tourism scenario. This study employs the Luwu tourism context (Latuppa Waterfall, Kambo Hill, Batara Guru Museum, and Palopo Peak Panorama) to stimulate students' reasoning activities. The fractions textbook created in this study is designed according to the RME technique, specifically within the context of Luwu tourism.

Prior research has not yet incorporated a fractions textbook that is methodically structured within a particular RME-based framework. Muslimin et al. (2020) employed a qualitative methodology to examine the incorporation of RME with Islamic principles. Nursyahidah et al. (2021) utilized a design study to investigate the amalgamation of RME with the Megono Gunungan tradition. Pathuddin et al. (2021b) employed a historical methodology to examine mathematics education within the framework of the traditional dish "Barongko." Risdiyanti and Prahmana (2021) employed design research to investigate RME-based instruction within the framework of Wayang and the Mahabharata narratives. Prahmana (2017) conducted an ethnographic study on mathematics education within the framework of Yogyakarta batik designs. These experiments illustrate the promise of context-specific RME implementations but do not yield a fractions textbook tailored to a specific local situation. This gap offers an opportunity to develop a fractions textbook centered on the RME technique in the context of Luwu tourism.

This study's originality is attributed to two primary characteristics. The study develops a fractions textbook based on the RME technique within the Luwu tourism environment, designed to enhance students' reasoning skills. Secondly, design research is utilized as the methodological foundation for the development and enhancement of the educational material. The originality of this work is evident in its specific output: a fractions textbook contextualized within the Luwu tourism framework, which has not been previously documented in research. The resultant textbook is anticipated to substantially contribute by acting as a reference for educators in instructing fraction themes and by enhancing theoretical understanding of the influence of local settings on mathematical reasoning.

The research question motivating this project is, How are the processes and consequences of building a fractions textbook aligned with the RME technique within the Luwu tourism environment intended to enhance students' reasoning? The primary aim of this work is to develop a fractions textbook contextualized within Luwu tourism to enhance students' reasoning skills. To tackle this research topic, we employed a design research methodology that included preliminary design, teaching experiments, and retrospective analysis.

2. METHOD

Design research reasoning guided this study's R&D design. A fractions textbook based on Realistic Mathematics Education (RME) in Luwu tourism was developed, refined, and validated. Three primary steps of R&D were preliminary design, two teaching experiment cycles (Cycle I and Cycle II), and retrospective analysis. Each

cycle tested the textbook in real classrooms, identified problems, and revised the design to improve students' mathematical reasoning.

The study was conducted at Primary School 5 Tarabbi Indah, South Sulawesi. One Grade V class of 23 pupils and their instructor participated. Both cycles used the same model teacher who applied the RME-oriented textbook. One expert professor from Universitas Cokroaminoto Palopo and one mathematics teacher from Primary School 5 Tarabbi Indah validated the textbook, test items, scoring rubric, and response questionnaires. The participants were chosen to ensure that the textbook was written and tested in an elementary school setting with real curriculum and classroom constraints.

The textbook was developed, validated, and evaluated using several tools. A Luwu tourism-based RME fractions textbook was the principal product under development. The textbook used contextual issues from Mesjid Jami Tua Palopo, the Luwu Royal Traditional House, Bukit Kambo, and Palopo City life to initiate fraction exercises. Contextual fraction questions in the textbook tested students' logic. Students' solutions were graded on analytic reasoning indicators such as interpretation, assessment, and inference. The rubric defined performance levels and descriptors for each indicator to evaluate students' thinking.

Expert validation sheets were created for the textbook, reasoning test items, analytic reasoning rubric, and teacher and student feedback surveys. The item validation sheets addressed content, construction, and language. The content aspect analyzed the indicators' alignment with the 2013 curriculum's fundamental competencies and each item's reasoning potential. The construction phase assessed item count, instruction clarity, and logical ordering. Language accuracy, punctuation, and conversational clarity were assessed. The rubric validation sheet examined rubric-item alignment, weighting clarity, descriptive statements for each indication, and accurate and clear language. Teachers and students completed questionnaires to assess the textbook's classroom usefulness. Experts used validation sheets to check question formulation and language use on both questionnaires. The teacher response questionnaire assessed task relevance to classroom needs, alignment with basic competencies, working time, clarity of instructions, and presentation. Nine items on the student response questionnaire asked students to rate clarity of instructions, legibility of text, attractiveness of design, usefulness of images, sufficiency of time, contextual relevance of the problems, contribution to fraction understanding, assessment of mastery, and motivational impact. On a four-point Likert scale, higher scores indicated more positive evaluations.

In both cycles, observation notes recorded classroom processes throughout instructional experiments. The observations focused on how students used the textbook, responded to circumstances, and solved fraction problems. The research followed the three design research processes in two iterations. In the preliminary design phase, researchers used document study and Focus Group Discussions (FGDs) with teachers, lecturers, and students to analyze needs. Most tests focused on remembering, comprehending, and applying, and fractions were rarely taught using genuine problems. Current textbooks did not thoroughly measure thinking. Based on these findings, the researchers created a Luwu tourism-based RME fractions textbook. An FGD linked

fraction subtopics to specific situations, such as Mesjid Jami Tua Palopo and Bukit Kambo tourism teaching fractions and mixed numbers. Expert validators reviewed a textbook, reasoning exam items, an analytic rubric, and a response questionnaire draft.

Cycle I used the validated textbook in class. The researcher observed the Grade V instructor teach fractions using the textbook based on RME. In class, students did contextual fraction activities and took the reasoning test. The teacher and students completed response surveys to assess the textbook and tasks' applicability after class. Researchers collected students' written work, questionnaire responses, and observation notes. Retrospective analysis followed Cycle I. To detect textbook problems, researchers examined expert validation data, students' reasoning scores, teacher and student replies, and classroom observations. The investigation found that restricted working time, confusing instructions, too broad a content breadth, and visual complexity that raised cognitive burden impeded reasoning. These observations led to four primary revisions: lengthening working time, enhancing item phrasing, reducing the scope to one core fraction issue, and simplifying visuals and layout.

Cycle II implemented the revised textbook in Grade V with the same class or an equivalent parallel class, depending on school regulation. The model instructor utilized the redesigned textbook in fraction classes again. The researcher monitored the implementation to ensure that lessons matched the specified scenarios and Cycle I conditions. Students took a second reasoning test with changed tasks and completed the student response questionnaire. Teachers completed the teacher response questionnaire again. All Cycle I and Cycle II data were obtained under similar conditions for meaningful comparison.

Data analysis was descriptive and comparative. To assess validity, textbook, item, rubric, and questionnaire expert validation scores were averaged for content, construction, and language. These average scores were compared to preset criteria to classify instruments as valid or very valid. Second, teacher and student response data were analyzed by computing cumulative and mean scores for each descriptor and instrument. Mean ratings were then categorized into practicality categories on a four-point scale from low to very practical. Students' reasoning test results were scored analytically. Total reasoning scores were tallied and divided into very good, good, fair, and poor categories for each student. Cycle I and Cycle II distribution tables showed student numbers and percentages. The textbook updates' effects on reasoning ability were examined by comparing these distributions.

Content analysis was used to analyze qualitative data from student writing, teacher questionnaire comments, and classroom observations. The investigation examined students' reasoning processes, context interpretation issues, justification completeness and structure, and contextual image utilization to support their ideas. These qualitative findings showed how textbook design changes like reduced images, clearer directions, and a tighter subject focus improved students' thinking throughout cycles.

3. RESULTS AND DISCUSSION

Results

Cycle I

In Cycle I we carried out three stages of activities. The first stage was the Initial Design, which included focus group discussions (FGDs) to analyse students' needs, determine the content and sequence of the material, develop the textbook and supporting instruments, and test the validity of the textbook. The next stages were the teaching experiment and the retrospective analysis.

Initial Design

FGD on Students' Needs Analysis, Determination of Content and Sequence, and Textbook Development

At this stage, supporting resources for an RME-oriented textbook to assess students' thinking were collected and documented. The researchers first examined the content domain and found fractions to be a key assessment area. Fractions, which had the lowest achievement rate, were chosen for the RME-oriented textbook to measure students' reasoning skills.

Next, we reviewed past research on reasoning instruments and textbook design theories. This review examined how RME-oriented textbooks measure students' fraction reasoning. Students today need thinking skills, according to the literature review. Next, data was collected on students, teachers, and the instruments teachers use to assess students' reasoning abilities through document analysis to set the instrument's goals. Teachers, professors, and students from Primary School 5 Tarabbi Indah participated. After gathering data, various conclusions emerged: (1) Teachers' textbooks did not yet assess students' mathematics learning outcomes comprehensively; (2) the cognitive abilities assessed mostly focused on remembering, understanding, and applying and did not fully involve reasoning ability; and (3) the textbooks, especially for fractions, lacked realistic problems.

The document analysis of teacher-used tests indicated that assessment tasks focused on students' process application and understanding. Few teachers tested students' abilities to analyze and evaluate, and cultural literacy test items were unusual. This suggests that pupils' mathematical literacy reasoning is poorly measured. Table 1 summarizes the student needs analysis.

Table 1. Students' Needs Analysis

No.	Aspect Examined	Findings
1	Assessment instruments developed by teachers	Do not yet measure students' reasoning ability
2	Teachers' understanding of textbooks and instruments to measure reasoning	Teachers do not yet understand instruments for measuring reasoning
3	Students' level of understanding and reasoning ability	Students' reasoning ability in solving fraction problems is still low
4	Teachers' understanding of the RME approach	Teachers have limited understanding of integrating RME into textbooks
5	General conclusion	There is a need to develop an RME-oriented textbook to stimulate students' reasoning

The Table 1, it can be concluded that: (1) The assessment instruments developed by teachers do not yet comprehensively measure students' reasoning ability; (2) Teachers do not yet understand the constructs needed to measure reasoning ability; (3) Students' reasoning ability is low; and (4) The textbooks currently used by teachers do not sufficiently stimulate students to reason. Based on these data, an analysis was then carried out to identify a measurement model capable of assessing students' critical thinking skills in junior high school mathematics, particularly in geometry. In summary, the results of problem and needs identification (the purpose of developing the instrument) are presented in the following Table 2.

Table 2. Problem Identification

Identified Problem	Need (Instrument Objective)
Assessment instruments developed by teachers do not stimulate reasoning ability, especially in fractions	Teachers need to develop instruments that can stimulate students' reasoning ability in mathematics, particularly in fractions
Teachers do not yet understand instruments that can stimulate reasoning ability	There is a need to develop instruments that are easy for teachers to understand but that can still stimulate students' reasoning ability
Students have low reasoning ability	The competencies to be developed and assessed should emphasise reasoning ability
Textbooks developed by teachers do not yet connect the material with the RME approach	There is a need to develop an RME-oriented textbook that can stimulate students' reasoning ability

Next, we conducted a Focus Group Discussion (FGD) to determine the sequence of topics and the RME contexts to be embedded in the textbook to be designed. The sequence of content is shown in the following Table 3.

Table 3. FGD Results

Topic	RME Content Description
Introducing fractions through the context of Mesjid Jami Tua Palopo	Students are introduced to fractions through a series of images taken from the Mesjid Jami Tua Palopo building
Introducing equivalent fractions through the context of the Luwu royal traditional house	Students learn about equivalent fractions through structural aspects of the Luwu royal traditional house
Introducing mixed numbers through the tourism context of Bukit Kambo	Students learn about mixed numbers through tourism activities at Bukit Kambo
Understanding fraction operations through contextual problems in Palopo City	Students understand fraction operations through problems based on situations in the community of Palopo City

Textbook Validity Testing

We created an item validation form for three assessment areas before designing the instruments. The content aspect assessed the items' indicators' compliance with the 2013 curriculum core competencies for fractions and their capacity to engage students' reasoning. The construction aspect evaluated the item number, instruction clarity, and presentation. The language aspect assessed grammar, punctuation, and clarity. We created a two-part logic scoring rubric validation form. Content included aligning the

analytic rubric with the product, clarifying the weighting mechanism, and describing each indicator. Language accuracy and clarity were assessed. The item validation form and scoring rubric validation form were reviewed by one expert lecturer from Universitas Cokroaminoto Palopo and one mathematics teacher from Primary School 5 Tarabbi Indah to verify the reasoning-stimulating items and rubric.

We also created validation forms for student and teacher response surveys, which included two assessment components. The language aspect assessed language accuracy and clarity, while the construction aspect assessed question meaning and intent. One expert lecturer from Universitas Cokroaminoto Palopo and one mathematics teacher from Primary School 5 Tarabbi Indah reviewed the student response questionnaire validation form to ensure that it was suitable for assessing the practicality of the developed tasks from the students' perspective. A mathematics instructor used the validation form for the teacher response questionnaire to assess the activities' applicability after one expert lecturer validated it. The teacher response questionnaire assessed two areas. The first concerned the objects' construction and content, including whether they met classroom demands, basic competencies, and completion time. The second concerned language and presentation, including instruction and task content clarity and task layout appeal. The student response questionnaire was given after students completed the exercises to assess their practicality. Two validators verified it before usage. It had eight questions about instruction clarity, visual presentation (text, graphics, and layout), and task usefulness after completion.

Teaching Experiment

Validation Results for Reasoning Test Items

The item validation form assessed by the expert validators included three aspects: content, construction, and language. Each aspect contained several descriptors used as the basis for scoring. The validation results from the expert lecturer and the mathematics teacher are presented in the Table 4 below.

Table 4. Validation Results for the RME-Oriented Textbook Items

No.	Descriptor	Validator 1 score	Validator 2 score	Average Descriptor Score	Average Aspect Score
	Item Content Aspect			**3.50**	
1	The item indicators align with the Core and Basic Competencies (KI/KD) in the 2013 Curriculum (revised 2018) for fractions	4	4	4.00	3.50
2	The developed items can stimulate students to reason	3	3	3.00	3.50
	Item Construction Aspect			**3.92**	
3	The number of items is appropriate for the students' needs on the topic of fractions	3	3	3.00	3.92
4	The items are appropriate for the topic of fractions	4	4	4.00	3.92

No.	Descriptor	Validator 1 score	Validator 2 score	Average Descriptor Score	Average Aspect Score
5	The instructions for working on the items are clear and easy to understand	4	4	4.00	3.92
6	The items are arranged systematically, clearly, and are easy to understand	4	4	4.00	3.92
7	The items can be used to identify reasoning in the aspect of interpretation	4	3	3.50	3.92
8	The items can be used to identify reasoning ability	4	4	4.00	3.92
9	The items can be used to identify reasoning in the aspect of evaluation	4	4	4.00	3.92
10	The items can be used to identify reasoning in the aspect of inference	4	4	4.00	3.92
	Language Use Aspect			**3.87**	
11	The items are stated clearly and are not ambiguous	4	4	4.00	3.87
12	The images in the items are clear and relevant to the problem	3	4	3.50	3.87
13	The language used in the items is communicative and easy to understand	4	4	4.00	3.87
14	Spelling and punctuation follow the rules of Indonesian orthography (EYD)	4	4	4.00	3.87
	Average score per validator	**3.78**	**3.78**		**3.78**

The validation results indicate an average score of 3.50 for the content aspect, 3.92 for the construction aspect, and 3.87 for the language aspect. The mean validation score was 3.78. Consequently, the created items were classified as very valid based on the established criteria.

Validation Results for the Reasoning Scoring Rubric

The scoring rubric validation form, assessed by the expert validators, included two aspects: content and language. Each aspect contained several descriptors used as the basis for scoring. The validation results are presented in the following Table 5.

Table 5. Validation Results for the Reasoning Scoring Rubric

No.	Descriptor	Validator 1 score	Validator 2 score	Average Descriptor Score	Average Aspect Score
	Content Aspect			**3.80**	
1	The construction of the analytic rubric is aligned with the developed items	3	4	3.50	3.80
2	The assessment aspects are based on reasoning ability	3	4	3.50	3.80
3	The weights of aspects and sub-aspects are appropriate for stimulating reasoning ability	4	4	4.00	3.80
4	The descriptors for each indicator clearly distinguish performance levels	4	4	4.00	3.80
5	The scoring rubric is arranged systematically, neatly, and clearly	4	4	4.00	3.80
	Language Aspect			**3.25**	
6	The descriptors follow the rules of Indonesian orthography (EYD)	4	4	4.00	3.25
7	Each descriptor statement is not ambiguous	3	4	3.50	3.25
	Average score per validator	**3.57**	**4.00**		**3.78**

The average score for the content aspect was 3.80, while for the language aspect it was 3.25. The mean validation score was 3.78. The scoring rubric for evaluating students' reasoning, previously designated as critical thinking, was consequently deemed valid according to the standards.

Validation Results for Student and Teacher Response Questionnaires

The validation forms for student and teacher response questionnaires, assessed by expert validators, also included two aspects: construction and language. Each aspect contained several descriptors used as the basis for scoring. The validation results for the student response questionnaire are presented below.

Table 6. Validation Results for the Student Response Questionnaire

No.	Descriptor	Validator 1 score	Validator 2 score	Average Descriptor Score	Average Aspect Score
	Question Construction Aspect			**3.80**	
1	The statements have clear meaning and purpose	4	4	4.00	3.80
2	The instructions for completing the questionnaire are clear	4	4	4.00	3.80

No.	Descriptor	Validator 1 score	Validator 2 score	Average Descriptor Score	Average Aspect Score
3	The statements are related to the topic of fractions	4	4	4.00	3.80
4	The questions can capture whether the allocated time is sufficient for students	3	3	3.00	3.80
5	The statements can assess the appearance and aesthetic value (attractiveness) of the tasks	4	4	4.00	3.80
Language Use Aspect				**4.00**	
6	The language used in the questionnaire is easy to understand and communicative	4	4	4.00	4.00
7	Spelling and punctuation follow Indonesian orthography (EYD)	4	4	4.00	4.00
	Average score per validator	**3.85**	**3.85**		**3.85**

The student response questionnaire obtained an average score of 3.80 for the construction aspect and 4.00 for the language aspect, giving an overall average validation score of 3.85. According to the criteria, the student response questionnaire for assessing product practicality was therefore classified as valid. After the items and scoring rubric for assessing students' reasoning (labelled as critical thinking in the original text) met the validity criteria and were revised based on the validators' suggestions, the items were tried out. Students' reasoning ability was assessed by scoring their work using the scoring rubric. The trial was conducted in two stages. Class V was selected based on the teacher's recommendation, as it was considered to have better ability than the other classes. The details of the textbook trial are shown below.

Table 7. Details of the RME-Oriented Textbook Trial

No.	Implementation Stage	Date
1	Stage I	Monday, 13 August 2023
2	Stage II	Monday, 20 August 2023

The trial was conducted with the involvement of the classroom teacher and the researcher as observers. At the beginning of the session, the researcher explained the technical aspects of working on the tasks and clarified the focus of the assessment. The classroom situation during the trial is illustrated in the following figure.

Figure 1. ***Classroom Conditions during the Trial***

After the trial of the RME-oriented textbook, both teachers and students completed response questionnaires to provide data on the practicality of the product. The results are described below.

Teacher Response Questionnaire Results

The following table presents the results of the teacher response questionnaire on the practicality of the RME-oriented textbook, assessed from the aspects of construction, content, language, and presentation.

Table 8. Teacher Response to Product Practicality

No.	Descriptor	Score
	Construction and Content Aspect	
1	The developed tasks help the teacher assess students' reasoning ability	4
2	The developed tasks are aligned with the topic of fractions	4
3	The developed tasks are aligned with Basic Competencies and Learning Objectives in the 2013 Curriculum (revised 2018)	4
4	The number of items developed matches the time needed for students to complete them	3
5	The developed tasks help students understand the material	4
6	The developed tasks are visually attractive and engaging for students	4
	Language and Presentation Aspect	
7	The visual appearance of the tasks is attractive and motivates students	4
8	The instructions for working on the tasks are clear and easy to understand	4
9	The language used in the tasks is communicative and easy for students to understand	4
10	The placement of images and task content is clear and not confusing	3
	Total Score	38
	Average Score	3.8

The average teacher response score was 3.8, which meets the criterion for practicality. In addition to scoring the questionnaire, the teacher also provided comments regarding the implementation of the RME-oriented textbook, which are summarised as follows: (a) The time allocated for students to work on the tasks was perceived as insufficient, partly because the trial coincided with the early phase of the return to face-to-face learning; (b) The teacher suggested correcting some typographical errors in the items; and (3) The scope of content was considered too broad and should be focused on a single main topic.

Student Response Questionnaire Results

The following table presents the results of the student response questionnaire (36 students) regarding the practicality of the product.

Table 9. Student Responses to Product Practicality

No.	Descriptor	Number of Students Choosing Each Option	Total Score
		1	2
1	The instructions in the tasks are communicative and easy to understand	1	1
2	The type, style, and size of the font used in the tasks are clear and appropriate	0	0
3	The tasks are designed attractively	0	1
4	The images in the tasks help me understand the problems	0	0
5	The time provided is sufficient to complete the tasks	0	3
6	The tasks are aligned with fraction problems and are contextual	0	0
7	The tasks help me understand the topic of fractions	0	1
8	The tasks assess my mastery of the material that I have learned	0	2
9	The tasks motivate me to study the material more seriously	0	0
	Total Score		
	Average Score		

The average student response score was 3.33, indicating that the developed product is practical. However, students suggested that more time be allocated to complete the tasks, as the given time was perceived as insufficient for solving all the problems. Overall, the student and teacher responses indicated: (a) Positive responses to the use of the textbook; and (b) Average scores that met the practicality criteria. Therefore, the developed RME-oriented textbook met the criterion of practicality.

Retrospective Analysis of Cycle I

Based on the initial design and teaching experiment, several conclusions can be drawn, as summarised in the following table.

Table 10. Summary of Testing Results in Cycle I

No.	Type of Test	Description of Findings
1	Practicality Test	The instruments used in the study were classified as valid based on expert validation
2	Effectiveness Test	The implementation of the textbook in Cycle I was categorised as effective

Cycle II

The Cycle I teaching experiment showed that the RME-oriented fractions textbook using the Luwu tourism context was valid and practical, however many areas needed reworking to improve students' reasoning. Data comes from students' written work, classroom observations, teacher and student response questionnaires, and learning

process and product retrospectives. Cycle II changes targeted four main areas. Initially, task time was changed. Students needed more time to research tourism, analyze photos, and apply fractions. Some Cycle I students couldn't form whole reasoning reasons. Students had more time to enhance interpretation, analysis, and arguments. Second, item wording improved. In Cycle I, some items contained typos, poor phrasing, or confusing directions. Rewritten phrase structure, contextual explanations, and procedural phases simplified RME instructions while retaining their qualities. Third, substance was scarce. Cycle I coverage was too broad, impeding students' reasoning. Cycle II taught one main topic in critical fractions. This technique helped teachers and students explore subjects and setting and structure their thinking. The layout and presentation were simplified. The Luwu tourist contextual visuals were maintained, however several were streamlined for functionality and cognitive ease. Instead of beautifying the page, the layout was changed to make important information easier to access and support reasoning. The new RME-oriented textbook for Cycle II teaching experiment meticulously incorporated these adjustments.

Table 11. Focus of Textbook Revisions in Cycle II

No.	Focus of Revision	Rationale for Revision	Form of Revision Implemented
1	Increasing time allocation for working on tasks	Students needed more time to read the contexts and construct reasoning justifications	Working time was extended, and instructions for working were clarified
2	Improving item wording	Typographical errors and ineffective sentences were found in Cycle I	Revision of command sentences, correction of typing errors, simplification of instructions
3	Narrowing the scope of material	The material was too broad, which fragmented students' reasoning focus	The textbook was focused on a single core fraction topic as a priority
4	Simplifying images and layout	Some images were too complex and did not directly support reasoning processes	Simplification of visuals, rearrangement of layout, emphasis on key information

The Cycle II teaching experiment tested the new RME-oriented textbook in Luwu tourism. The implementation at Primary School 5 Tarabbi Indah maintained consistency in procedures and classroom settings as in Cycle I to credit any variations in outcomes to the textbook amendments. Depending on school policy and research design, Cycle II participants were either the same Grade V students as Cycle I or a similar class. This selection maintained student comparability in initial skills and learning experiences to allow for more valid cycle comparisons.

In Cycle II, the Grade V classroom teacher was the model instructor and used the redesigned textbook for RME-based fraction education. The researcher observed and co-facilitated classes to ensure they followed learning scenarios and instructional requirements. Each session used the same textbook, student worksheets, reasoning rubric, and instructor and student response questionnaires. Classroom setting, seating, visual media, and support were consistent. This uniformity was necessary to attribute Cycle II learning gains to the updated textbook rather than extraneous influences.

Reasoning Ability Results

Cycle II's reasoning test uses updated RME textbook. The study found that most of the 23 students comprehended Luwu tourism contexts in fraction problems and linked contextual information to fraction concepts. Students' reasoning improved from Cycle I. In Cycle II, 16 students (69.6%) were good or very good, up from 11 (47.8%) in Cycle I. Students were able to express their reasoning, interpret visuals, and draw conclusions from settings. The responses grew contentious. Instead of duplicating computations, most students wrote better structured arguments than in Cycle I. Students improved most in interpretation and evaluation, explaining linkages between material in Mesjid Jami Tua Palopo, the Luwu Royal Traditional House, Bukit Kambo tourism, and ordinary Palopo City contexts. Understanding these settings improved reasoning. Below is Table 12 with Cycle II students' reasoning scores.

Table 12. Distribution of Students' Reasoning Ability

Reasoning Category	Number of Students	Percentage
Very Good	6	26.1%
Good	10	43.5%
Fair	5	21.7%
Poor	2	8.7%
Total	**23**	**100%**

Figure 2. *Distribution of Students' Reasoning Ability*

Teacher Response Results

The Cycle II teacher response questionnaire demonstrated that the Luwu tourism-focused RME textbook was more practical and easier to use than Cycle I. Teacher praised clear directions, proper language use, and thorough contextual information that helped pupils understand fractions in real life. Alignment with Basic Competencies, task presentation, and the textbook's reasoning guidance received high marks out of ten descriptors. Improved item wording, visual simplification, and subject narrowing helped the teacher focus lessons and improve reasoning processes. Teacher awarded an average score of 3.9 on a 4-point scale, which is practical. This suggests that Cycle II modifications improved the textbook's usability and convenience of use for fractions instruction.

Table 13. Teacher Response Results in Cycle II

No.	Descriptor	Score
	Construction and Content Aspect	
1	The developed tasks help the teacher assess students' reasoning ability	4
2	The developed tasks match the topic of fractions	4
3	The tasks developed align with the Basic Competencies and Learning Objectives in the 2013 Curriculum (revised 2018)	4
4	The number of items matches the time available for students to complete them	4
5	The developed tasks help students understand the material	4
6	The developed tasks are visually attractive and engaging for students	4
	Language and Presentation Aspect	
7	The tasks have an attractive appearance that motivates students	4
8	The instructions for working on the tasks are clear and easy to understand	4
9	The language used in the tasks is communicative and easy for students to understand	4
10	The placement of images and task content is clear and not confusing	3
	Total Score	39
	Average Score	3.9

Student Response Results

The 23 Cycle II students loved the Luwu tourism-based RME textbook. Overall, student answers improved from Cycle I. Most students scored 3–4 on all descriptions. Twelve to sixteen kids scored three and seven to fourteen earned four for each item. Students found the instructions straightforward. Text display, font size, and task layout were satisfactory. The textbook's photos of Mesjid Jami Tua Palopo, Luwu Royal Traditional House, and other tourist spots explained fraction problems. Most pupils liked Cycle II's working time better.

Students found the exercises relevant and tied to real-life situations, making the information easy to grasp. Students claimed the tasks assessed their knowledge and motivated them to study. Due to improved answer quality, the new Cycle II textbook scored 1,267 and averaged 3.44, making it practical. Studies suggest that textbook enhancements including graphic simplicity, better directions, and more focused settings improve Luwu tourist students' fraction understanding and learning experiences. Table 14 provides questionnaire results.

Table 14. Student Response Results for Product Practicality in Cycle II

No.	Descriptor	Number of Students Choosing Each Option	Total Score
		1	2
1	The instructions in the tasks are communicative and easy to understand	0	0
2	The type, style, and size of the font used in the tasks are clear and appropriate	0	0
3	The tasks are designed attractively	0	0
4	The images in the tasks help me understand the problems	0	0
5	The time provided is sufficient to complete the tasks	0	0
6	The tasks are aligned with fraction problems and are contextual	0	0

No.	Descriptor	Number of Students Choosing Each Option	Total Score
7	The tasks help me understand the topic of fractions	0	0
8	The tasks assess my mastery of the material I have learned	0	0
9	The tasks motivate me to study the material more seriously	0	0
	Total Score		
	Average Score		

Retrospective Analysis Results in Cycle II

Cycle II retrospective analysis assessed how successfully the revised RME-oriented textbook using Luwu tourism improved students' thinking and solved Cycle I flaws. The reasoning exam scores, instructor response questionnaire, student response questionnaire, and classroom observations inspired this reflection. The analysis indicated Cycle II enhanced learning quality more than Cycle I.

Students' reasoning improved after the textbook makeover. Very good or very good thinking test scores rose from 47.8% in Cycle I to 69.6% in 23. Students improved their logical reasoning, contextual links, and fraction inferences. The highest gains were in two indicators. First, evaluate context, find relevant data, and link contextual images. Evaluation involved strategy selection and justification. Clarifying issues, enhancing lesson structure, and reducing visual load improved students' reasoning.

Cycle II teachers rated the new textbook 3.9 out of 4 for practicality. The teacher noticed several major changes from Cycle I: the textbook became more focused due to a narrowed content scope; simplified visuals made context explanations easier; clearer instructions supported reasoning rather than calculating; and Grade V Luwu tourism tasks were relevant and easy to use. The textbook also helped Cycle II pupils construct stronger thinking reasons, according to the teacher.

Cycle II student response questionnaire scores averaged 3.44, making the textbook practical. Students praised the textbook's clarity, font size, layout, and pictures; the Luwu tourism context's relevance for fraction learning; the assignments' capacity to assist conceptual comprehension rather than computation; and the familiar and meaningful circumstances, which motivated them. Many Cycle I students complained about work. Most students liked Cycle II's time allocation, while others requested more. So Cycle II's time adjustment worked. Table 15 highlights Cycle II's retrospective analysis's main findings.

Table 15. Retrospective Analysis Results in Cycle II

No.	Component Analysed	Findings on Cycle I	Revision in Cycle II	Results in Cycle II	Implications
1	Students' reasoning ability	Many students wrote only computational procedures; reasoning justifications were	Increased working time, clearer instructions, simplified visuals, narrower	The proportion in good–very good categories increased to 69.6%. Answers became more	Revisions successfully improved reasoning. The textbook became more effective in

No.	Component Analysed	Findings on Cycle I	Revision in Cycle II	Results in Cycle II	Implications
		incomplete. The proportion in good–very good categories was only 47.8%.	content focus.	argumentative and complete.	facilitating reasoning through local contexts.
2	Understanding of Luwu tourism context	Some students were confused by overly complex images.	Simplified images, improved layout, more concise contextual explanations.	Students understood the context more quickly and accurately. Misinterpretations decreased.	Simple contextual visuals improved the accuracy of students' interpretations.
3	Clarity of task instructions	Some instructions were ambiguous and too long.	Revised wording, shorter sentences, clearer directions.	Students more quickly understood what to do. Complaints about instructions decreased.	Clear and concise instructions improved response quality and reduced cognitive load.
4	Scope of material	The material was too broad; time was insufficient and students' focus was scattered.	Narrowed the scope to a single core fraction topic.	Instruction became more focused, allowing deeper exploration of concepts.	Limiting the scope of content increased the depth of students' reasoning.
5	Textbook practicality (teacher)	The teacher rated the appearance as good but found some tasks difficult to guide within the available time. Average score: 3.8 (practical).	Improved presentation, more coherent sequencing of tasks, reduced visual load.	Average score increased to 3.9 (very practical). The teacher found it easier to facilitate students' reasoning.	The textbook supported instruction and helped the teacher monitor students' reasoning more effectively.
6	Textbook practicality (students)	Average score: 3.63 (practical). Working time was still insufficient.	Increased time allocation and improved task presentation.	Average score: 3.44 (still practical). Time was perceived as more adequate; the layout was easier to read.	The textbook was accepted by students and helped them understand fractions through local contexts.
7	Learning motivation	Some students did not yet show interest	Simplified narratives and	Motivation scores increased across the nine	The local Luwu context was effective in

No.	Component Analysed	Findings on Cycle I	Revision in Cycle II	Results in Cycle II	Implications
		in the context used.	strengthened the relevance of local contexts.	questionnaire items; more students chose the highest rating (4).	enhancing students' emotional engagement and motivation.
8	Time-use effectiveness	Working time was insufficient, and answers were incomplete.	Increased duration and shortened task wording.	Students were able to complete the tasks with more complete reasoning.	Time planning emerged as a key factor in eliciting mathematical reasoning.

Discussion

Cycle II results demonstrate that the redesigned fractions textbook, based on the Realistic Mathematics Education (RME) strategy in Luwu tourism, improved students' reasoning skills and the product's practicality for instructors and students. These findings suggest that a well-structured textbook with realistic and local contexts can improve mathematical thinking better than procedural exercises. Tasks and a didactical framework in the textbook guided students from contextual problems to explicit reasoning about fractional relationships.

Cycle II students' thinking improved, suggesting the adjustments were not just technical but affected how they approached the problems. More pupils improved their reasoning performance, identifying relevant facts, choosing acceptable strategies, and providing logical mathematical arguments. The core principle of RME is that mathematical reasoning develops when students work with problems that are close to their lived experiences and require interpretation, analysis, and tentative formalization rather than direct recall of procedures (Gravemeijer & Van Eerde, 2009; Van den Heuvel-Panhuizen & Drijvers, 2020). Luwu tourism provided relevant settings for students in this study. This realization may have helped develop more stable mental representations of fractions, making part–whole relations and proportions easier to reason about.

Cycle II design changes clearly addressed cognitive load. In cycle I, most students successfully interpreted graphics with many details and extensive explanation words; thus, they split their focus between context decoding and solution preparation. Cycle II rewrote contextual descriptions in fewer phrases and simpler images to highlight mathematically relevant facts. According to cognitive load theory, this reduction in superfluous load allowed pupils to focus more on mathematical structural analysis rather than decoding details (Ginns & Leppink, 2019; Kirschner et al., 2018). Student responses became more argumentative and structured, and they exhibited clearer linkages between contextual information and mathematical concepts.

The realistic situation mediated informal and formal understanding, as shown by interpretation and assessment gains. Students proceeded from horizontal mathematization, where they ordered and represented the real scenario, to vertical

mathematization, where they acted on and modified their mathematical models, when they meaningfully comprehended tourism circumstances. This movement is important to RME and directed innovation. Students did more than apply fraction manipulation principles in this study. They reinvented fractional amounts, equivalence, and proportionality by studying Luwu tourism ticket prices, distances, and sharing circumstances. In cycle II, reasoning improves due to procedural fluency and conceptual development from better instructional design.

Student solutions' representational patterns changed in Cycle II. Many students used verbal explanations, basic diagrams, tables, and symbolic notation to support their claims instead of describing computing procedures. This shift from computational to reasoned reactions that combine many representations is crucial. As part of progressive mathematization in RME, students are coordinating fractional relationship expressions. Flexible switching between contextual explanations, visual models, and symbolic fractions is considered essential for mathematical understanding. These findings show that the textbook design, which explicitly encouraged students to describe and reflect on their thoughts in multiple ways, fostered this flexibility.

Beyond these conceptual improvements, cycle II reasoning improvement suggests that the new textbook reduced various epistemological difficulties from cycle I. Reduced image misinterpretations, irrelevant tactics, and fractional value-concrete referent links. This pattern shows that the redesigned activities and images provided better didactical phenomenology by selecting real-world situations that naturally elicited the mathematical structures. In this study, Luwu tourism was more than a backdrop. They gave students opportunities to compare quantities, reason about parts and wholes, and make proportion-based judgments, which are essential to fraction understanding. Students understood mathematical regularities better when these occurrences were shown in crisper images and focused prose.

The cycle II study shows that students' reasoning improved more than just from repeated exposure to similar tasks. It was directly related to textbook didactical design improvements that aligned with RME concepts. Students developed higher-order mathematical thinking more sustainably using local tourism contexts, explicit cognitive load attention, and structured horizontal and vertical mathematization. These findings suggest that context-based RME textbooks, when carefully designed, can support elementary students' fraction reasoning and provide a practical pathway for teachers and curriculum developers who want to integrate local contexts into fraction instruction without sacrificing conceptual depth and reasoning goals.

4. CONCLUSION

The fractions textbook for Realistic Mathematics Education (RME) in Luwu tourism is valid, practical, and excellent at stimulating students' reasoning. Experts evaluated the textbook as highly valid for content, item construction, language, and contextual alignment. This means the material, problems, and reasoning rubric meet theoretical and pedagogical requirements. Both cycles' instructor and student answers were practical to highly practical. Revision improved task presentation, instruction clarity, and contextual

alignment in cycle II. Teachers said the textbook was straightforward to use and helped assess students' reasoning, while students said the contextual questions helped them learn fraction ideas. Effectiveness increased kids' reasoning skills. Good and very good students increased from 47.8% in cycle I to 69.6% in cycle II. Students improved in computational techniques, context interpretation, information evaluation, and mathematical explanations. These findings demonstrate that genuine situations from students' lives improve conceptual understanding and higher-order thinking. The updated textbook design in cycle II addressed cycle I's problems in clarity, visual load, time efficiency, and material depth. Iterative R&D and reflection helped the textbook become a contextual mathematics resource for elementary schools.

Several limitations must be considered in this investigation. The number of research participants was limited because only one class was involved in each cycle; therefore, generalizations should be made with caution. Second, the textbook's performance depends on students' initial abilities and the teacher's instructional style; therefore, this study doesn't properly represent teacher and school heterogeneity. Third, the textbook's Luwu tourist background is particularly relevant for pupils in a particular region, but it may vary in schools with diverse cultures. Fourth, the study included just two cycles, so long-term learning outcomes improvement could not be fully assessed. Even while experts verified the researchers' analytic rubric, scoring may be subjective because students' reasoning was analyzed using it.

This study has major consequences for primary mathematics education. The study suggests that the RME technique with local settings can improve students' mathematical reasoning. According to RME's guided reinvention principle, realistic and relevant real-world situations enable students to grasp fraction concepts more concretely and transcend from informal to formal knowledge. The produced textbook can help teachers create more relevant, higher-order thinking-focused contextual teaching resources. These findings can help schools create local-culture-based curricula and train teachers in RME implementation. For researchers and curriculum developers to create legitimate, practical, and effective teaching materials, the study emphasizes an iterative process of development, trial, revision, and reflection. Thus, the study helps construct an RME-based fractions textbook and provides an empirical foundation for learning resource design in different contexts and content areas.

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