

Mental Image and its Impact on Types of Errors in Mathematical Problem Solving: A Case Study

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ABSTRACT

Students' difficulties in visualizing geometric objects often trigger various errors in problem-solving. Although mental imagery has an important function in clarifying the shape, position, and relationships between geometric elements, research specifically examining its relationship with student error types is still limited. This study aims to describe the relationship between mental imagery and student error types in solving geometric problems. The research method used is a qualitative approach with a case study design. The research subjects consisted of three mathematics education students selected based on the variety of errors they made. Data was collected through geometry tests, observations, think-alouds, and in-depth interviews. The results showed that inaccurate or incomplete mental imagery led to the emergence of visual representation errors, conceptual errors, procedural errors, and calculation errors. Mental imagery that is not integrated with conceptual and procedural understanding causes students to misinterpret problems. The contribution of this study emphasizes the importance of strengthening visualization skills and integrating geometric concepts in learning to minimize students' cognitive errors.

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1. INTRODUCTION

Mathematics is frequently regarded as a formidable discipline for numerous students. Resolving mathematical problems needs both comprehension of abstract concepts and the capacity to visualize and portray information in accessible formats, such as graphs, diagrams, geometric figures, or variable relationships (Pongsakdi et al., 2020; Verschaffel et al., 2020). A critical cognitive element in this process is mental imagery. Mental imagery denotes an individual's capacity to conjure images or information internally without direct external stimulation (Park & Yoo, 2020; Pearson et al., 2015). In the realm of mathematics education, mental imaging pertains to the ability of pupils to perceive geometric entities, diagrams, graphs, or interrelations among mathematical

concepts mentally before or during problem-solving activities (Bates et al., 2021; Fastame, 2021).

Studies indicate that mental imagery facilitates the connection between visualization and problem-solving skills (Zaleskiewicz et al., 2023). Research by Garderen has shown that visual representations markedly enhanced student performance in resolving mathematical word problems (van Garderen et al., 2021). Schematic imagery usage correlated positively with achievement, but diminished pictorial imagery levels correlated with inferior performance (Van Garderen et al., 2018). Moreover, the research conducted by Douville and Pugalee indicates that mental imagery strategies can enhance mathematical problem-solving abilities in both elementary and secondary school students (Douville & Pugalee, 2003; Nuovo et al., 2018), as well as in prospective mathematics educators (Kirmac & Bulut, 2013; Rifat, 2018; Zulu, 2023). This suggests that visual and mental imagery training in mathematics education can alleviate problem-solving challenges and enhance performance.

Conversely, inaccuracies in mathematical problem-solving can present in multiple forms, including computational errors (Permatasari et al., 2021), conceptual inaccuracies (Esterlina et al., 2023; Inganah et al., 2021), and errors in visual representation or mental modeling of problems (Inganah et al., 2021; Zakaria et al., 2010). A literature study indicated that student errors arise not solely from a deficiency in content comprehension but may also stem from an inadequate use of mental imagery. For instance, when students struggle to visualize a three-dimensional item or fail to relate a diagram to a real-world context, it impedes the problem-solving process (Prayekti et al., 2020). This study will be done to ascertain whether these issues also manifest among pre-service mathematics teachers regarding geometry concepts. Consequently, comprehending the function of mental imagery is crucial in the realm of mathematics education, especially in discerning how mental visualization may assist or impede pre-service mathematics educators in resolving geometry issues.

Most previous research has focused on the general correlation between mental imagery and achievement (Commodari et al., 2024). This study offers a new perspective by constructing a taxonomy of geometry errors specifically caused by malfunctioning mental imagery processes in pre-service teachers, going beyond the analysis of procedural or computational errors commonly studied. Unlike previous studies that focused on elementary or secondary school students (Guarnera et al., 2019), this study explores the internal cognitive mechanisms of pre-service mathematics teachers. Its novelty lies in revealing how mental imagery barriers at the tertiary level remain a fundamental factor in the persistence of geometric conceptual errors. Furthermore, this study introduces a qualitative analysis model that integrates mental imagery theory with Newman's or Radatz's error analysis to detect the precise point at which contextually irrelevant mental representations interfere with mathematical decision-making in geometry topics.

This study seeks to investigate the influence of mental imagery on the errors committed by pre-service mathematics teachers when solving mathematical tasks. This study will employ a qualitative approach to examine the viewpoints of pre-service

mathematics teachers about their utilization of mental imagery in addressing geometry problems and its correlation with the errors they commit. This study will address several primary questions using interviews, observations, and analysis of the performance outcomes of prospective mathematics teacher students. In what manner do prospective mathematics teacher students conceptualize the challenges they encounter, and are these inaccuracies associated with difficulty in constructing precise or contextually relevant mental representations? This study aims to offer new insights into the significance of cultivating mental visualization abilities in mathematics education and to furnish recommendations for educators to devise more effective teaching ways to mitigate errors associated with mental imagery.

2. METHOD

This study utilized a qualitative methodology, including a case study design. The objective was to investigate comprehensively how student mathematics teachers employ mental images in resolving geometric problems and how this visualization skill correlates with the types of errors they commit. The participants were chosen by purposive sampling and comprised numerous students from the Mathematics Education Study Program at STKIP PGRI Situbondo. The selection of subjects was predicated on the variety of errors encountered in geometric problem solving, guaranteeing representation of each error type by the students. The study comprised three students, each exhibiting at least one error type from the following categories: visual representation errors, conceptual errors, procedural errors, interpretation errors, and calculation errors. This methodology enabled the researcher to undertake a more thorough investigation of the attributes and origins of each error type, while also elucidating the function of mental imagery in the error process.

Data was gathered by direct observation of each subject's geometric problem-solving methodology, comprehensive interviews, think-aloud techniques, and review of student documentation. Observations were conducted to record visualizing behavior, encompassing the propensity to sketch, the use of language that conveys mental imagery, and the manifestation of imaginative manipulations of geometric objects. Semi-structured interviews were conducted to investigate students' experiences in creating and utilizing mental images, as well as to examine how these visualizations contributed to errors. A think-aloud technique was employed to document the students' cognitive processes and the development of mental imagery during the resolution of geometry problems.

All data underwent thematic analysis, encompassing data reduction, coding, thematic pattern grouping, and comprehensive interpretation (Naeem et al., 2024; Sitasari et al., 2022) of the correlation between mental visualization capacity and error categories for each participant, as illustrated in Figure 1. Data validity was ensured by triangulation methods, integrating observation, interviews, and think-aloud protocols.

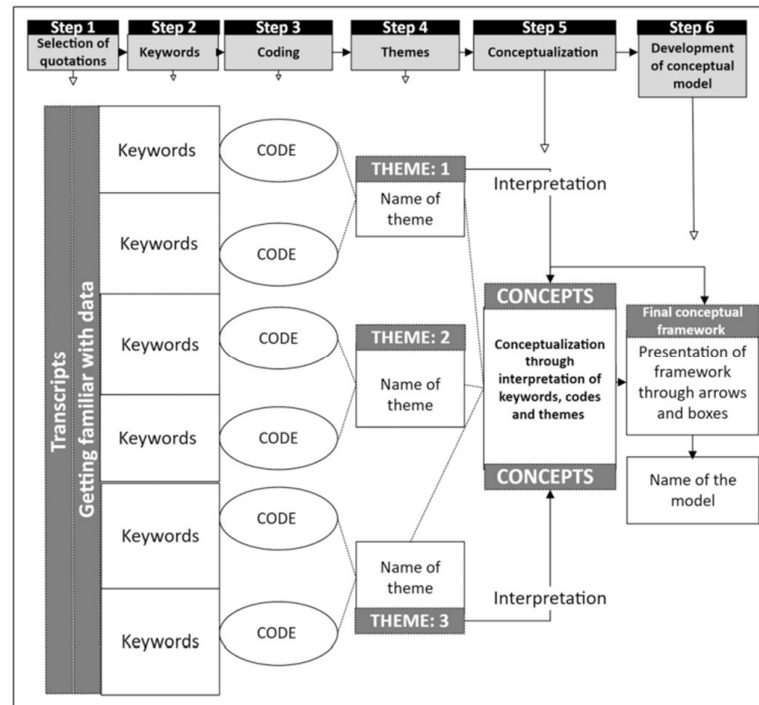


Figure 1. Thematic Data Analysis Process (Naeem et al., 2024)

This study also adheres to ethical research principles, where each student was given clear information about the research objectives and procedures before giving consent to participate. Subject identities were kept anonymous, and all data obtained were used solely for academic research purposes. With this methodology, the study is expected to provide an in-depth understanding of how mental imagery influences the emergence of various types of errors in geometry problem solving, so that the findings can be used as a basis for improving learning strategies for prospective mathematics teachers.

3. RESULTS AND DISCUSSION

Results

This study involved three Mathematics Education students at STKIP PGRI Situbondo, selected based on the types of errors they encountered in solving geometric problems. The three subjects exhibited distinct error characteristics, allowing the researcher to analyze the relationship between mental imagery and the types of errors they made. Data was obtained through geometry problem-solving tests, observations of the work process, think-alouds, and in-depth interviews.

Subject 1 (S1): Visual Representation Errors and Problem Interpretation Errors

Findings on Visual Representation

Subject 1 (S1) had trouble in accurately visualizing geometric shapes. He depicted a sphere as a disproportionate circle, as seen in Figure 2, and took a relatively long time to complete.

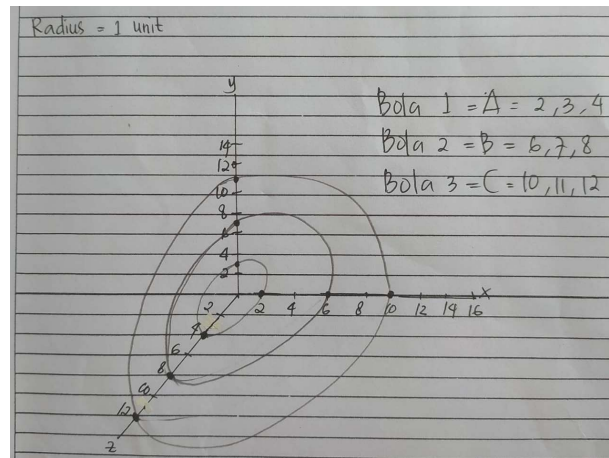


Figure 2. Visual representation of S1

Throughout the think-aloud session, the participant consistently remarked, "It appears that the ball resembles this," showing ambiguity over the mental representation of the shape. In the interview, S1 remarked about his image, "There are three balls, A, B, and C arranged in this manner." In response to inquiries regarding the association between the coordinates (2,3,4), (6,7,8), and (10,11,12) and each ball, S1 remarked, "*The balls traverse these coordinates*," while indicating his illustration.

Findings on Question Interpretation

S1 comprehended the issue by interpreting the provided coordinates as points traversed by the balls, as indicated by his assertion, "The balls pass through these coordinates." To evaluate the validity of this assertion, the researcher subsequently inquired about the centroid of each sphere. S1 asserted, "The center is located at the midpoint of this circle."

S1's interpretation of the coordinate positions (2,3,4), (6,7,8), and (10,11,12) as trajectories of the ball is coherent and is evidenced by the results of the distance computations he conducted, as illustrated in Figure 3.

Figure 3. Calculation of the shortest distance between balls by S1

Subject 2: Conceptual and Calculation Errors

Conceptual Errors Found

Subject 2 was able to visually represent the problem quite well, as shown in Figure 4. This aligns with his explanation during the interview, namely, "*I imagined three balls at a certain distance*."

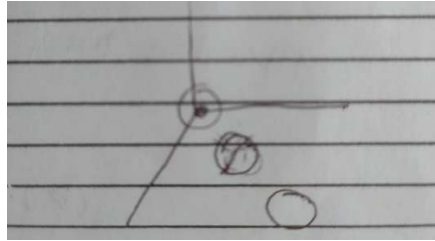


Figure 4. Visual representation of S2

When drawing, S2 stated, "The center of ball A is at two-point three point four," and then proceeded to determine the coordinates of each center point. When calculating the distances between balls A and B and C, the distances can be seen in Figure 5.

$$AB = \sqrt{(6-2)^2 + (7-3)^2 + (8-4)^2}$$

$$= \sqrt{16 + 16 + 16}$$

$$= \sqrt{48} = 4\sqrt{3}$$

$$BC = \sqrt{(10-6)^2 + (11-7)^2 + (12-8)^2}$$

$$= \sqrt{16 + 16 + 16}$$

$$= \sqrt{48} = 4\sqrt{3}$$

Figure 5. Misconceptions and Calculations by S2

When asked about the shortest distance, S2 stated, "The shortest distance is the distance from A to B," while pointing to the calculation result for the distance from A to B.

Findings of Calculation Errors

S2 seemed to be hasty in performing the math and resolved the problem more swiftly than other participants. The computation executed by S2 in Figure 5 indicates that S2 concluded $(6-2)^2 = 3^2$. The researcher inquired whether S2 have sufficient confidence in the calculation's accuracy. S2 asserted with confidence, "Yes, I am certain of my answer," after a brief review of the calculation from beginning to end. Subsequently, when the researcher attempted to elicit S2's math error by inquiring, "What were you envisioning while calculating $(6-2)^2$?", S2 promptly responded, "Oh my God, yes, it was incorrect, sir." I instinctively envisioned the calculation of three minus two from six.

Subject 3: Procedural Error

Subject 3 envisioned a linear arrangement of three spheres. He remarked, "To determine if any of them intersect, I may initially compute their distance," articulated in a notably anxious and hesitant manner. The outcomes of S3's computation are illustrated in Figure 6.

$$\begin{aligned}
 1. & (6-2) + (7-3) + (8-4) \\
 & = 4 + 4 + 4 \\
 & = 12 \quad \} B-A \\
 \\
 & A-B = (10-6) + (11-7) + (12-8) \\
 & = 4 + 4 + 4 \\
 & = 12 \\
 \\
 & C-A = (10-2) + (11-3) + (12-4) \\
 & = 8 + 8 + 8 \\
 & = 24 \\
 \\
 2. & \text{bola } AB = 12 - 2 = 10 \\
 & \text{bola } BC = 12 - 2 = 10 \\
 & \text{bola } AC = 24 - 2 = 22
 \end{aligned}$$

Figure 6. Procedural Error by S3

During the interview process, S2 said shyly, "Sorry sir, I forgot how to calculate the distance of the ball."

Discussion

The findings of this study indicate that mental imagery plays a crucial role in developing geometric problem-solving strategies and can also be a source of errors when the mental image is inaccurate.

The Role of Mental Imagery in Errors in Visual Representation and Problem Interpretation (S1)

Subject 1 demonstrated that inaccurate visual representations led to errors from the initial stages of the problem-solving process. Mistakes in drawing individual spheres led to errors in identifying the spheres' shape and location, as well as in interpretation, demonstrating that a blurry or disproportionate mental image directly impacts subsequent errors. This finding aligns with several studies that suggest that erroneous visual imagery can impair mathematical problem-solving ability (Fastame, 2021; Kirmac & Bulut, 2013; Rifat, 2018; Van Garderen, 2006; Zulu, 2023).

Subject 1's misinterpretation of the problem when interpreting the center point as the point through which the spheres pass demonstrates that intuitive mental imagery is often a source of misconceptions when not aligned with mathematical definitions. This supports Presmeg's theory that spontaneous visualization needs to be linked to conceptual structures to avoid misinterpretation (Schoenherr & Schukajlow, 2024). Similarly, Mudaly explains that students' success in problem-solving is also greatly influenced by their ability to connect mental images with formal representations, such as reasoning and conceptual understanding (Mudaly, 2021).

Conceptual and Calculation Errors Resulting from Inaccurate Mental Images and Overconfidence (S2)

Subject 2 committed a conceptual error due to forming an inaccurate mental image. He imagined the distance between two balls by calculating the distance between their

centers. This indicates that S2 used the correct concept of distance between balls, namely by subtracting the distance between the centers from their respective radii. The choice of mental images based on convenience, rather than geometric truth, suggests that mental images can lead to misconceptions. This finding aligns with [Douville and Pugalee \(2003\)](#), who emphasized that visualizations should be built on conceptual understanding, not intuition alone. Subject 2's error clarifies that mental images serve not only as aids but can also trigger conceptual errors if students do not fully understand the properties of shapes.

Mental imagery is a visual or cognitive representation formed in the mind when facing a particular problem or task ([Blackwell, 2021](#)). In this case, a person's mental image of numbers and calculations influences how they approach a mathematical problem ([Bates et al., 2021](#)). For example, someone might imagine a simpler or easier number without examining each step of the calculation.

In this study, S2, when faced with the operation $(6-2)^2$, automatically presented the number 3 as the result of $(6-2)$. This automaticity was evident in S2's speed in performing the calculation. This occurred because S2 was more familiar with the combination of the numbers 6, 3, 2. Therefore, when presented with the stimulus of the numbers 6 and 2 in the form $(6-2)$, the number 3 appeared in his mental image more often than the number 4. This is clarified by S2's statement that he imagined the number 3 in his mind when subtracting 6 from 2.

S2's error occurred only with the combination 6, 3, 2. He did not appear to make errors with other number operations. For example, in a series of operations $(6-2)^2 + (7-3)^2 + (8-4)^2$, S2 correctly calculates $(7-3)^2$, which is 4^2 , and $(8-4)^2$ also produces 4^2 . In another number operation, when calculating the distance between the center point B and C (in this study, S2 interpreted it as the distance between ball B and C), $(10-6)^2 + (11-7)^2 + (12-8)^2$ produces the correct result, which is $4^2 + 4^2 + 4^2$. This indicates that inaccurate spontaneous mental images can lead S1 to errors in calculations ([Murphy, 2019](#)).

S2's calculation errors also appear to be influenced by excessive self-confidence, known as overconfidence. Overconfidence exacerbates the impact of inaccurate mental images. When S2 feels very confident that they know how to solve a problem or calculation, they may feel no need to double-check each step. This self-confidence makes S2 feel like they are "right" and ignore the fact that there is a possibility that they are making mistakes.

In the example of calculating the distance $(6-2)$, overconfident S2 may have felt they had performed the calculation correctly without double-checking, even though their result was clearly incorrect (assuming 3 instead of 4). Due to their overconfidence, they didn't feel the need to check whether the numbers they used were correct, and furthermore, they felt there was nothing wrong with their result, even though it was clearly incorrect. This contributed to S2's failure to recognize the error. In other words, overconfidence then led S2 to not check the error, because they already felt "certain" that they had solved the problem correctly. Only after receiving a stimulus related to their error did S2 realize their mistake.

Failure to Integrate Mental Imagery and Mathematical Procedures (S3)

Subject 3 demonstrated that even though the visual representation was correct, failure occurred when the mental imagery was not connected to the correct mathematical procedure. This finding supports the theory that visualization must be integrated with symbolic reasoning to be effective (Çakır & Stahl, 2012; Macchitella et al., 2023).

S3 imagined three balls lined up in different positions. The general procedure for answering questions on the geometry test was also depicted in his mental image. However, when he broke down the general procedure into a more specific procedure, namely determining the distance between balls, S3 failed. He seemed to spend quite a long time considering how to do it, before finally giving up, with the calculation results being ineffective.

This research enriches the cognitive psychology literature in mathematics education by, among other things, (1) Mapping Causal Relationships: In-depth explanation of how failures in mental image formation are the root cause of certain types of errors (e.g., conceptual understanding errors vs. transformation errors). (2) Validation of Dual-Coding Theory: Providing new empirical evidence regarding the importance of internal visual representations in supporting verbal-symbolic information processing at a more complex cognitive level.

Furthermore, teachers can use the results of this research as a guide. Teachers no longer simply teach formulas but focus on improving how students "see" problems mentally before writing answers. It provides recommendations for curriculum developers to integrate mental imagery exercises (such as mental rotation or visualizing diagrams without drawing) as part of problem-solving reinforcement. For the research subjects (prospective teachers), these results help them become aware of their visualization barriers, so they can be more careful when transferring abstract concepts to students in the future.

4. CONCLUSION

This research indicated that mental imagery plays a crucial role in the geometry problem-solving process. The mental images that students create in their heads affect how they understand problems, show geometric objects, choose concepts, and come up with ways to solve them. Accurate mental imagery is a key factor in successful problem-solving, while errors in visualization can lead to various other errors. First, this research found that visual representation errors occur when students' mental images are inaccurate, incomplete, or inconsistent with the actual structure of the geometric figure. Incorrect visual representations not only result in incorrect images on paper but also influence subsequent thought processes, resulting in inaccurate solutions. Second, conceptual errors arise when students form simplified mental images or those based on intuitive assumptions that are inconsistent with the properties of geometric figures. This investigation implies that mental visualizations lacking a solid conceptual foundation can result in misunderstandings and the application of unsuitable formulas. Third, this study shows that even accurate mental images don't guarantee that a problem will be solved correctly if they aren't used with the right math steps. The disconnect between

mental images and solution steps can lead students to choose the wrong strategy, erroneously organize calculation steps, or use theorems without a proper basis. Furthermore, calculation errors often stem not from arithmetic operations per se, but from a misconception about the size, shape, or geometric relationships being imagined. Overall, this research confirms that mental images can serve as a highly effective cognitive aid but can also be a source of error if the images formed are inaccurate, not supported by strong concepts, or not integrated with appropriate mathematical procedures. Therefore, the formation of correct, complete, and conceptual mental images is crucial for improving students' geometry problem-solving abilities.

As a suggestion, further research is recommended to develop or test specific learning models (such as Visual Thinking Routines or Dual Coding Theory-based learning models) that explicitly aim to train students' mental imagery skills before entering the formal calculation stage. Given the gap between mental imagery and actual geometric properties, further research could focus on the use of dynamic geometry software (such as GeoGebra or Cabri). Future research could expand the scope of the subject by considering differences in cognitive styles (such as visualizer vs. verbalizer) or gender differences. This is important to see whether error patterns in mental imagery are universal or influenced by the cognitive characteristics of certain individuals. Moreover, experimental research is essential to develop remediation programs that emphasize the alignment of visualization and conceptual comprehension. The focus is on how to transform incorrect intuitive assumptions in students' minds into formal understandings that are in accordance with geometric structures.

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